A Kinetic Model of Phytoplankton Growth and Its Use in Algal Control by Reservoir Mixing[1]

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Accelerated eutrophication (nutrient enrichment) caused by human activities has become a common feature of many natural and man-made lakes. The accompanying side effects, mostly due to increased primary production, are highly undesirable. Although reduction of the nutrient load (especially phosphorus) is generally considered most effective in opposing eutrophication, it will not always be an economic or practical solution.

Nutrient reduction is not feasible if most of the nutrient burden comes from diffuse sources (soil leaching) or if the reservoir has a very irregular flow regime, a condition that develops for lowland water supply reservoirs with pumped storage, such as those under construction in different European countries. A reliable in-reservoir method of algal control is of primary importance, since many of these reservoirs will draw on nutrient-rich river water.

It is thought that under certain circumstances turbulent mixing in a layer of sufficient depth might provide such a method of control. Although the relationship between natural turbulent mixing and algal growth is not very prominent in limnological studies, it is long since known in oceanography, the quantitative formulation being in Sverdrup's 'critical depth' theory. The effect of mixing on phytoplankton populations was confirmed in many destratification experiments in freshwater impoundments, most of which resulted in a decrease of the standing crop.

The object of this paper is to explain this effect by relating the algal growth rate to the mixing depth and the optical characteristics of the water.

MATHEMATICAL TREATMENT

Throughout this treatment the basic assumptions are that the algal distribution stays homogeneous, even while the growth rate is increasing, and that the nutrients are not acting as limiting factors. The generally used equation for gross photosynthesis per unit of time beneath a unit of surface is modified by introducing a self-shading factor $n\epsilon_p$:

$$P_{\text{gross}} = n \cdot \frac{p_{\max}}{n\epsilon_p + \epsilon_w} \cdot F(i) \qquad (1)$$

in grams of carbon per square meter per hour, where

- n, number of algal cells per cubic meter;
- $p_{\max}$, maximum photosynthesis per cell per hour, grams of carbon;
- ϵ_p, specific extinction coefficient of one algal cell per cubic meter;
- $n\epsilon_p$, total extinction per meter caused by photosynthesizing algae, which is the self-shading factor;
- ϵ_w, average extinction coefficient per meter of water including turbidity and color;
- $F(i)$, dimensionless function of the light intensity

If one assumes that respiration per cell is independent of depth, the total respiration R per unit of time in a water column of unit surface can be calculated and can be expressed as a fraction of $p_{\max}$:

$$R = n \cdot rp_{\max} \cdot z_m \qquad (2)$$

in grams of carbon per square meter per hour, where r is the respiration coefficient and z_m is the depth of the water column in meters.

The net rate of photosynthesis in a water column with a depth z_m is then given by:

$$P_{\text{net}} = P_{\text{gross}} - R = np_{\max}\left(\frac{F(i)}{n\epsilon_p + \epsilon_w} - rz_m\right) \qquad (3)$$

If production is considered on a 24-hour basis, P_{gross} must be corrected for the 'photosynthetic day-length,' and the respiration coefficient can be adapted accordingly.

[1] This paper is presented in abstracted form.

From the boundary condition $P_{net} = 0$, $n = A$, and

$$z_m = F(i)/(A\epsilon_p + \epsilon_w)r \qquad (4)$$

i.e., a maximum algal concentration exists for every given mixing depth, other factors being constant.

If the optimal generation time T ($=m/p_{max}$, m being the carbon content of the algal cell) is introduced and both sides are divided by z_m, (3) can be rearranged to an equation of growth:

$$dn/dt = kn(A - n)/(B + n) \qquad (5)$$

in which

$$k = r/T = rp_{max}/m$$

$$A = [F(i)/z_m r\epsilon_p] - B$$

$$B = \epsilon_w/\epsilon_p$$

The solution of this equation is

$$B \ln \frac{n_t}{n_0} - (A + B) \ln \frac{A - n_t}{A - n_0} = Akt \qquad (6)$$

A small computer program has been written to calculate growth curves with the mixing depth z_m and the extinction coefficient ϵ_w as parameters. The other factors were assumed to be constant. The values chosen were $k = 0.1$/day, $r = 0.1$, $T =$ 1 day, $F(i) = 2.75$, and $\epsilon_p = 2 \times 10^{-11}$ m². Figures 1 and 2 show some of the results for $\epsilon_w =$ 0.2/m and $\epsilon_w = 0.4$/m (log 10 base), respectively.

DISCUSSION

A number of factors that play a role in algal production (e.g., grazing by zooplankton, adaptation to low light intensities, nutrient limitations, and the temperature dependency of photosynthesis and respiration) are neglected or are assumed to be constant, whereas they are not (light intensity $F(i)$, optimal generation time T, and specific algal extinction ϵ_p). The resulting smooth curves resemble very much the experimental growth curves found in algal cultures under standardized conditions. It must be understood that prediction or description of a natural planktonic cycle is not the intention of the model in its present form. Still some useful conclusions that are confirmed by practical observations can be drawn.

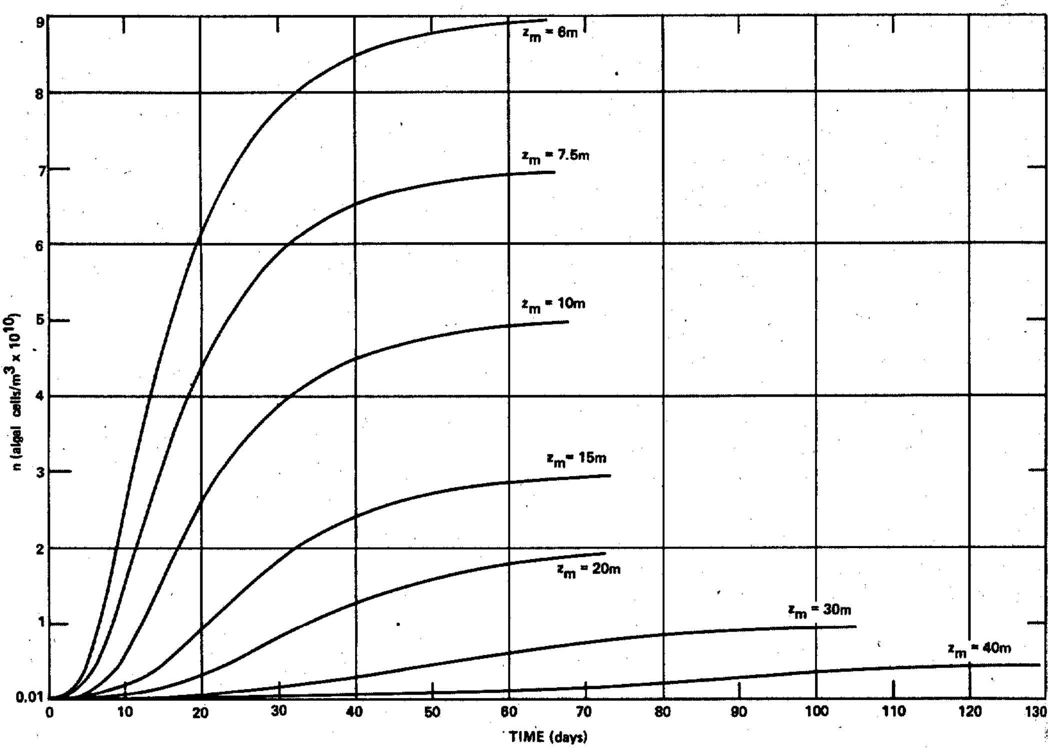

Fig. 1. Effect of mixing depth on growth for $\epsilon_w = 0.2$/m, $n_0 = 10^8$/m³, $k = 0.1$/day, $F(i) = 2.75$, and $\epsilon_p = 2 \times 10^{-11}$ m².

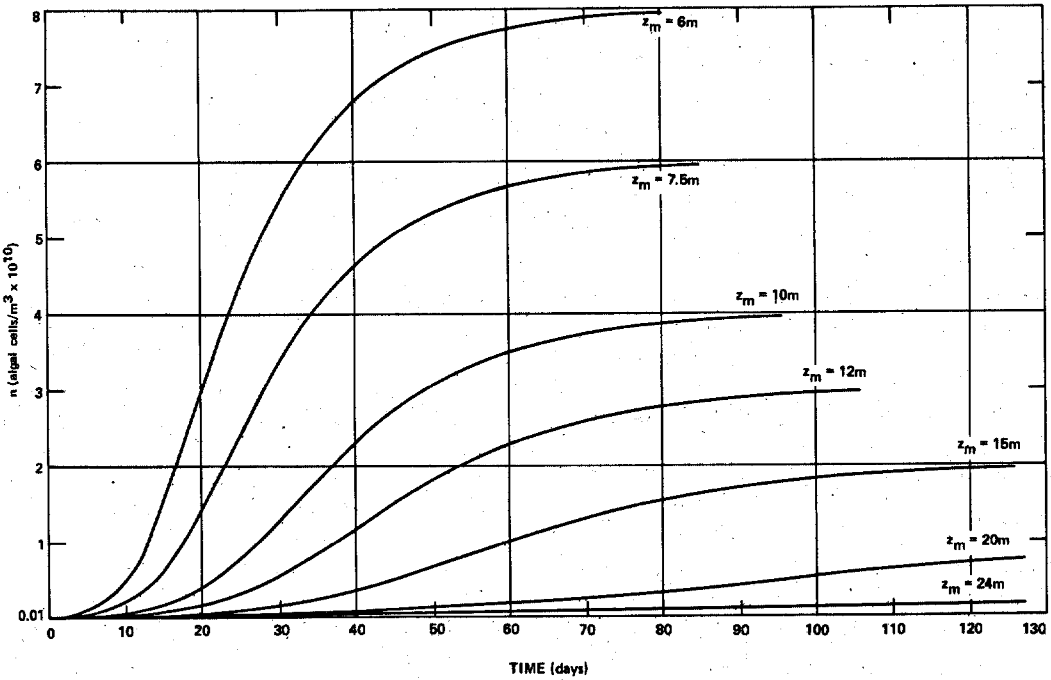

Fig. 2. Effect of mixing depth on growth for $\epsilon_w = 0.4$/m, $n_0 = 10^8$/m^3, $k = 0.1$/day, $F(i) = 2.75$, and $\epsilon_p = 2 \times 10^{-11}$ m^2.

The model predicts that relative productivity decreases as standing crop increases, a fact borne out in many studies of primary productivity. The highest assimilation rates per unit of biomass are often found in clear nutrient-poor oligotrophic lakes.

Water with a high natural turbidity and color will be less productive. Both the saturation level and the growth rate are lower than they are in clear water under comparable conditions.

The decrease in standing crop in many destratification experiments can be ascribed to a deepening of the mixed layer. Upward transport of an otherwise limiting nutrient from nutrient-rich lower layers can explain the increase recorded in some cases.

The conclusion that mixing depth can play a decisive role in algal production in cases in which only light is limiting seems justified. Whether this principle has a practical application for reservoir management will depend on the optical characteristics of the water itself. As an example the lowland reservoirs of the Biesbosch Water Supply Company, filled with water from the Rhine or Meuse, will serve.

At storage this water has an extinction coefficient ϵ_w (log 10 base) >0.4/m, which means that the photic zone will have a depth of <5 meters.

From present knowledge (Figure 2) the conclusion that a depth of 20–25 meters of mixed water would restrict algal growth to an acceptable level seems attractive. Maintaining a homogeneous distribution of algae over such a depth requires a continuous level of turbulence, which must be introduced artificially.

CONCLUSION

Many reservoirs subject to accelerated eutrophication will develop blooms of algae with associated undesirable side effects. Certain conditions being fulfilled, an increase in mixing depth might have a favorable influence on algal development. The use of reservoir mixing as a possible method of algal control merits attention.

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