RATIONAL MULTIVARIATE ALGAL GROWTH KINETICS

by

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INTRODUCTION

In general, results are limited that show the relationship between growth rates and nutrient concentrations that can be applied to mathematical models of the eutrophication process. The application of these few results is further complicated because of the different assay techniques and environmental conditions employed by various laboratories. If such independently collected data are to be used, there should be some common base to which all results can be converted. Between various laboratories, two environmental parameters that most frequently differ in algal assay results are light intensity and temperature. An attempt is made in the following sections to modify the most commonly used expression to describe algal growth kinetics to correct for temperature and light intensity.

KINETICS OF ALGAL GROWTH

The formulation most frequently used to show the relationship between growth rate and nutrient concentration in algal assays is the Michaelis-Menten (1913), or Monod (1949), expression

$$\mu(S) = \frac{\hat{\mu}(S)\, S}{K_S + S} \qquad \text{[light constant]} \qquad (1)$$

where

$\mu(S)$ = specific growth rate $\left(\frac{\text{gm cells produced}}{\text{gm cells-day}}\right)$, time^{-1}

$\hat{\mu}(S)$ = maximum specific growth rate as a function of nutrient concentration, time^{-1}

K_s = nutrient concentration at one-half the maximum specific growth rate, mass per unit volume

S = rate limiting nutrient concentration, mass per unit volume.

The maximum growth rate and the K_s value have been reported by Monod (1950, 1942) to be constants for a particular nutrient and culture. This expression has been used very successfully, but it must be remembered that it relates the effect of a single nutrient on the growth rate. In situations involving complex media and mixed cultures, the expression may apply only over limited ranges and many environmental factors also may influence the growth rate.

LIGHT INTENSITY AND ALGAL GROWTH RATE

It has been shown by Shelef et al., (1968) that the relationship between light intensity and the specific growth rate of algae is closely approximated by a hyperbolic function similar to the Monod equation above. That is

$$\mu(\ell) = \frac{\hat{\mu}(\ell)\ \ell}{K_\ell + \ell} \qquad \text{[nutrient constant]} \tag{2}$$

where

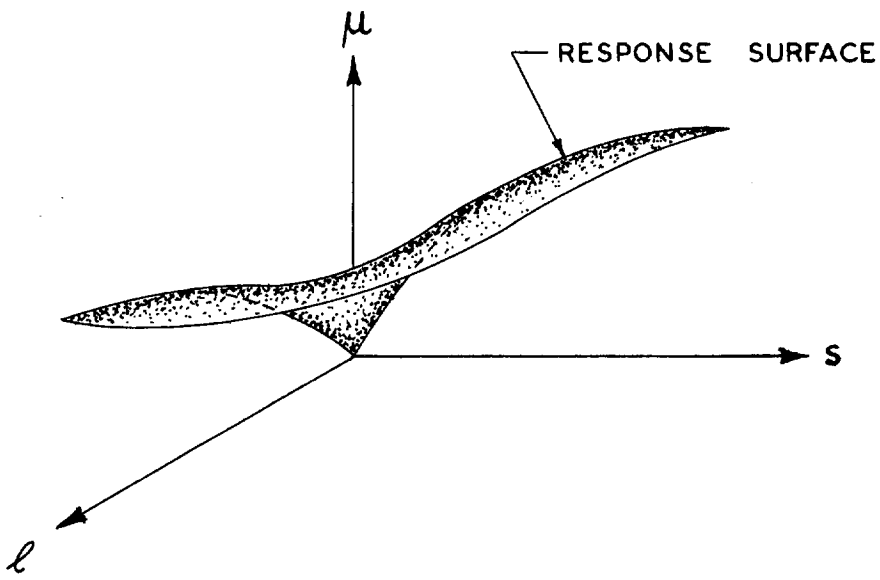
$\mu(\ell)$= specific growth rate for given nutrient concentration, time^{-1}

$\hat{\mu}(\ell)$= maximum specific growth rate attainable at given nutrient concentration, time^{-1}

ℓ = solar radiation or light intensity, langleys or calories/cm^2

K_ℓ = light intensity at one-half the maximum specific growth rate attainable (k), calories/cm^2.

Therefore, $\hat{\mu}(S,\ell)$ equals the maximum growth rate when S is a maximum and ℓ is optimum, or as shown below the growth rate is a function of the response surface.



Now, perhaps;

$$\mu(S,\ell) = \hat{\mu}(S, \ell) \frac{S}{K_S + S} \cdot \frac{\ell}{K_\ell + \ell} \qquad (3)$$

and when $S << K_s$ and $\ell << K_\ell$, the first order equation becomes,

$$\mu(S,\ell) \tilde{=} \frac{\hat{\mu}(S,\ell)}{K_s \cdot K_\ell} \cdot S \cdot \ell \cdot \qquad (4)$$

Equation 4 becomes our "model" for testing.

TEMPERATURE AND ALGAL GROWTH RATE

Algal growth rate, as in all biological reactions, is a function of temperature. A simplified temperature function which applied to many chemical reactions and the biochemical oxygen demand (BOD) test may be applicable to algal growth. This is

$$\frac{\mu_1}{\mu_2} = Z^{(T_1 - T_2)} \qquad (5)$$

where μ_1 and μ_2 are the specific growth rates at temperature T_1 and T_2, respectively, and Z is the temperature coefficient (Phelps, 1944). Based upon results from independent studies as adopted by Streeter and Phelps (1925), the best mean value for Z when used in BOD relationships is 1.047. Because of the lack of temperature-growth rate data for algal systems, the value will be assumed to be 1.047 for the analyses reported herein.

MODIFICATION OF THE BASIC EQUATION

The temperature coefficient can be incorporated into the basic equation as follows:

$$\mu(S,\ell,T) \tilde{=} \frac{\hat{\mu}(S,\ell,T_2)}{K_S \cdot K_\ell} Z^{(T_1 - T_2)} S \ell \qquad (6)$$

where Z is about 1.047 and T_2 is 20°C.

Further modification is possible using Equation 1 and assuming $K_\ell << \ell$ in Equation 2 which yields

$$\mu(S,\ell) = \frac{\hat{\mu}(S,\ell)\ S}{K_\ell\ (K_S + S)}$$

$$\frac{1}{\mu(S,\ell)} = \frac{K_\ell\ K_S}{\hat{\mu}(S,\ell)\ S} + \frac{K_\ell\ S}{\hat{\mu}(S,\ell)\ S}$$

$$\frac{1}{\mu(S,\ell)} = \left(\frac{K_\ell\ K_S}{\hat{\mu}(S,\ell)}\right) \frac{1}{S} + \left(\frac{K_\ell}{\hat{\mu}(S,\ell)}\right) .$$

Correcting $\mu(S,\ell)$ to 20°C yields $\mu(S,\ell,20) = \frac{\mu(S,\ell)}{Z^{T-20}}$.

Therefore,

$$\frac{Z^{T-20}}{\mu(S,\ell)} = \left(\frac{K_\ell\ K_S}{\hat{\mu}(S,\ell)}\right) \frac{1}{S} + \left(\frac{K_\ell}{\hat{\mu}(S,\ell)}\right)$$

$$\frac{Z^{T-20}}{\mu(S,\ell)} = \left(\frac{K_S}{\hat{\hat{\mu}}}\right) \cdot \frac{1}{S} + \frac{1}{\hat{\hat{\mu}}} \quad (\hat{\hat{\mu}} = \hat{\mu}/K_\ell) . \qquad (7)$$

The above mathematical model neglects many factors that can influence a biological system; however, it does incorporate two often neglected

variables and it is possible that the equation will partially explain deviations between results from various investigators.

RESULTS AND DISCUSSION

To demonstrate compliance with, or deviation from, the theoretical model presented in Equation 7, it is necessary to calculate values for the "constants" K_s and $\hat{\mu}$. These values can be determined by plotting the values for the left-hand side of the equation as the ordinate and the reciprocal of the nutrient concentration as the abcissa.

Steady-state data obtained by McGauhey et al. (1969) are plotted in Figures 1 and 2. These data were obtained from chemostats with a capacity of ~ 8,000 ℓ which were designed to simulate the shallow portions of Lake Tahoe. Lake Tahoe water used in the assays was pumped directly from the lake by means of a centrifugal pump located near the shoreline. The sample containing the enriching nutrient to be evaluated by assay was then pumped into the chemostats at a uniform rate appropriate to produce and maintain the desired concentration of sample in the LTW. Light intensity and temperature were monitored continuously.

Figure 1 shows the results obtained when the sum of the nitrate and ammonia nitrogen concentrations of secondary effluent from the South Tahoe Public Utilities District (STPUD) Wastewater Reclamation Plant was assumed to be the limiting nutrient. Other forms of nitrogen and the phosphorus concentrations were plotted for the secondary effluent sample, but all were poorly correlated which indicates that the nitrate and ammonia nitrogen were limiting the growth.

Figure 2 shows the results obtained when the orthophosphate concentra-

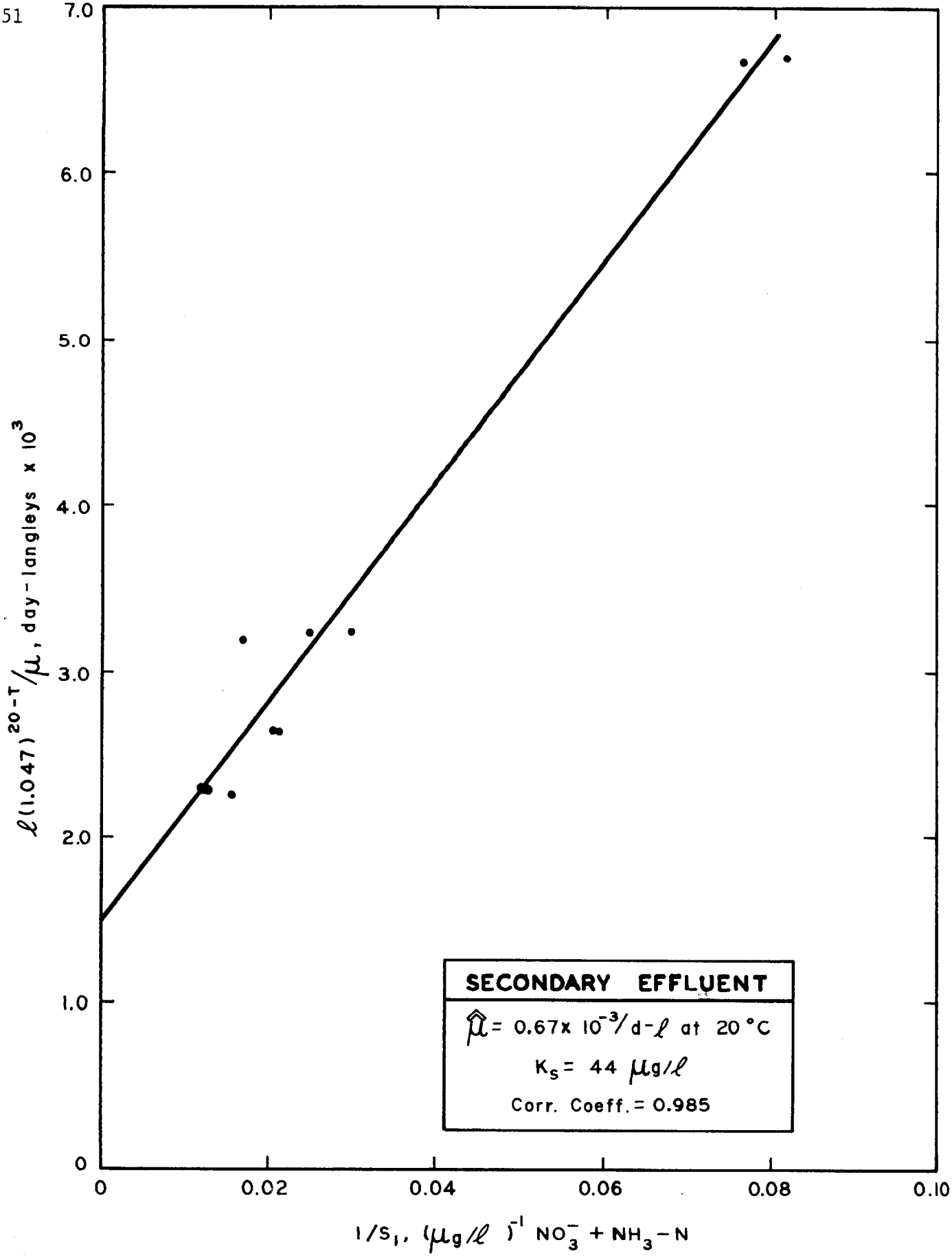


FIGURE 1. PLOT OF EQUATION (7) ASSUMING THAT THE SUM OF THE NITRATE AND AMMONIA NITROGEN CONCENTRATIONS REPRESENT THE LIMITING NUTRIENT IN THE SECONDARY EFFLUENT

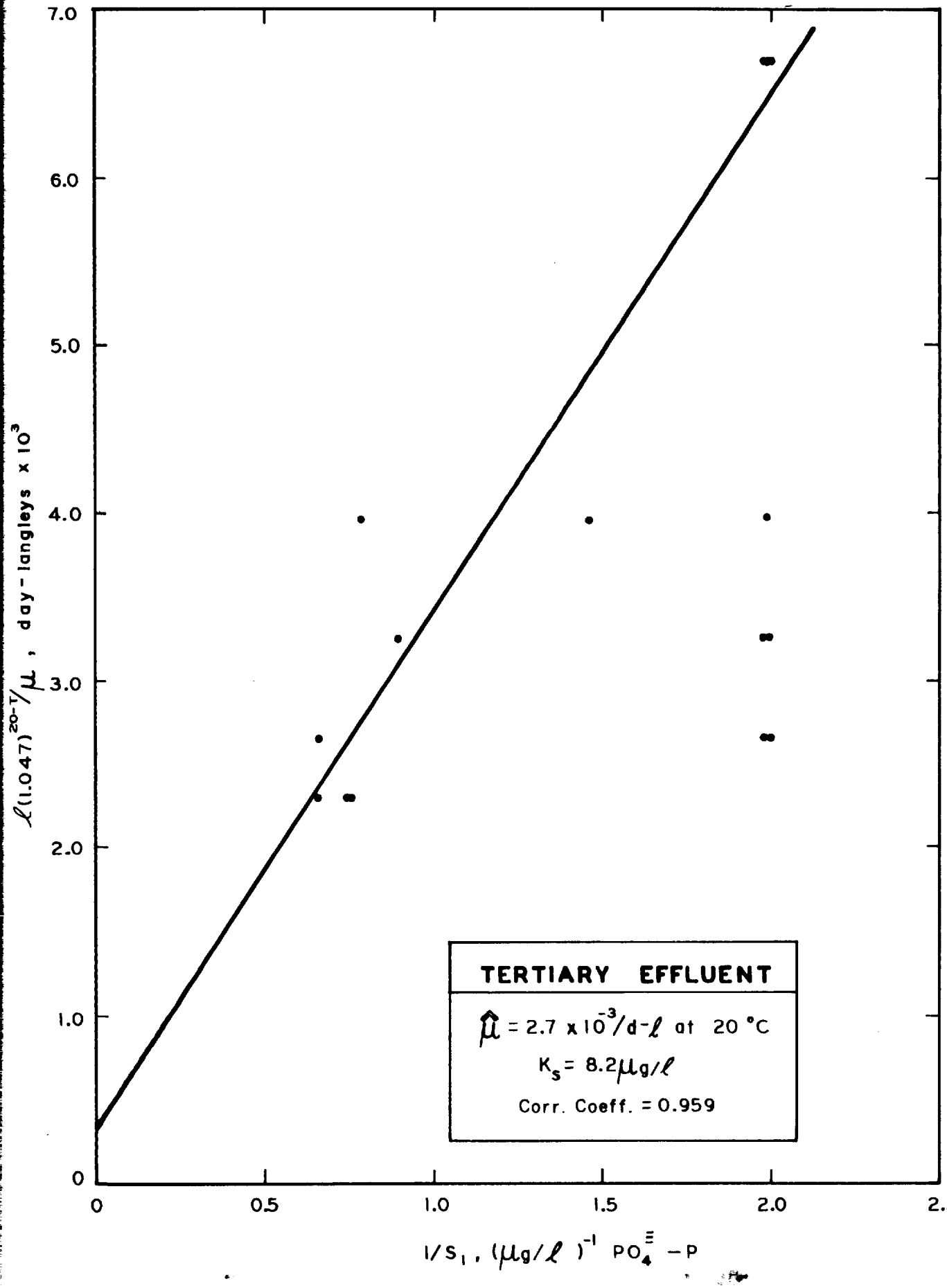


FIGURE 2. PLOT OF EQUATION (7) ASSUMING THAT THE ORTHOPHOSPHATE CONCENTRATION REPRESENTS THE LIMITING NUTRIENT IN TERTIARY EFFLUENT

tion (expressed as phosphorus) of tertiary effluent (Phosphorus removal only) from the STPUD plant was assumed to be the limiting nutrient. The regression analyses results shown on the figure do not include the five values for $1/S_1$ that equal 2.0 which fall well below the regression line. These were excluded because they represent the limits of the analytical procedure which was limited to 0.5µg/ℓ of orthophosphate as phosphorus. It is also very likely that the phosphate was exhausted long before the full potential for growth was achieved; thereby, resulting in a very low growth rate although the hydraulic residence time was adequate for a much greater response.

DISCUSSION AND CONCLUSIONS

The above results are only preliminary; however, it appears that the model describes data collected under natural environmental conditions even though the diurnal variation of light intensity and temperature were neglected and mean values for the period of steady-state operation were used in the equation. Additional data are being collected from various experiments performed at different temperatures and light intensities to determine if the model can be used to explain differences reported by various researchers. In view of the above results, it appears reasonable to expect the model to compensate for temperature and light intensity variations. Refinement of the temperature coefficient (Z) should further improve the model.

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