Metric Details for Natural-Language Spatial Relations*

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Abstract

Spatial relations often are desired answers that a geographic information system (GIS) should generate in response to a user's query. Current GISs provide only rudimentary support for processing and interpreting natural-language-like spatial relations, because their models and representations are primarily quantitative, while natural-language spatial relations are usually dominated by qualitative properties. Studies of the use of spatial relations in natural language showed that topology accounts for a significant portion of the geometric properties. This paper develops a formal model that captures *metricdetails* for the description of natural-language spatial relations. The metric details are expressed as refinements of the categories identified by the 9-intersection, a model for topological spatial relations, and provide a more precise measure than does topology alone as to whether a geometric configuration matches with a spatial term or not. Similarly, these measures help in identifying the spatial term that describes a particular configuration. Two groups of metric details are derived: *splitting ratios* as the normalized values of lengths and areas of intersections; and *closeness measures* as the normalized distances between disjoint object parts. The resulting model of topological and metric properties was calibrated for sixty-four spatial terms in English, providing values for the best fit as well as value ranges for the significant parameters of each term. Three examples demonstrate how the framework and its calibrated values are used to determine the best spatial term for a relationship between two geometric objects.

Categories and Subject Descriptors: H.2.3 [**Database Management**]: Languages—*query languages*; H.3.3 [**Information Storage and Retrieval**]: Information Search and Retrieval—

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General Terms: Design, Human Factors

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1 Introduction

The interaction between users and a geographic information system (GIS) has recently received increased attention (Mark and Frank 1992; Medyckyj-Scott and Hearnshaw 1993; Nyerges *et al.* 1995). It is often argued that GISs that better capture human behavior would be easier to use. The mode of interaction with which people are most familiar with is natural language, but GIS user interfaces with natural-language components are found rarely (Mark and Gould 1991). Current GISs only inadequately address spatial queries that are based on human intuition, as the main focus of today's implementations is on queries that are based on a *quantitative* representation. Such GISs are useful in answering metric-based queries, which involve precise angular and distance measures, but they cannot handle the way people communicate and interact through a *qualitative* language. For example, people only rarely give directions in precise details as in, "the grocery shop is 34.95 meters due east," but rather provide these instructions qualitatively as in, "the grocery shop is two blocks down the road, on the right" (Hernández 1994). GISs that are flexible enough to accommodate such human interpretation are expected to find a wider audience and user community than current systems that generally require a GIS specialist as a user.

This paper is concerned with the formalization of people's use of spatial relations in natural language. Formalizations of the semantics of natural-language spatial relations such that they can be represented within a GIS, are necessary to allow for qualitative spatial queries. In that respect, this research lies at the heart of *Naive Geography*, the field of study that is concerned with formal models of the common-sense geographic world (Egenhofer and Mark 1995b). Formal models currently used in commercial GISs lack the parameters to accommodate the flexibility that natural language has in partitioning space. Knowledge about the parameters that play a significant role in the selection of spatial predicates by people in describing spatial relations will allow us to develop appropriate formal models and to calibrate them to fit human intuition.

In the past, the semantics of spatial relations have received attention primarily in the arena of linguistics and artificial intelligence. Clark (1973) suggested a strong correspondence between perceptual space (P-Space), which humans use to perceive the space around them, and linguistic space (L-Space), which is used by language to represent the perceived space. This correspondence has been widely used in subsequent research for eliciting natural language descriptors of scenes. Talmy's (1983) seminal paper, "How Language Structures Space" establishes the link between prototypical spatial configurations and the use of natural language predicates. A variety of properties may contribute to the choice of a particular spatial term, such as the natural language (English vs. Spanish), the culture, the semantics of the spatial objects involved, the tasks users envision, the context in which the objects are presented, the pictorial presentation (a sketch vs. a topographic map), and the objects' geometries (Mark *et al.* 1995). We concentrate here on the *geometric* aspects, giving preference to metric properties (such as length and area measures) and topological properties of spatial relations (such as coincidence and containment) over explicit influences of cardinal directions (such as angles) and shapes.

The approach taken here is a refinement of the 9-intersection (Egenhofer and Herring 1991) to accommodate more semantics of natural-language spatial relations. It builds on strong evidence that among those spatial relations, topological properties are most fundamental (Piaget and Inhelder 1967; Lynch 1960; Kuipers 1978; Riesbeck 1980; Mark and Egenhofer 1994a; Regier 1995), and that metric properties are appropriate refinements of certain topological configurations. For instance, for a road to *enter* a park, it was found that it is critical that the road crosses the park's boundary (Mark and Egenhofer 1994b); however, additional metric properties, such as the portion of the road that is inside the park, may also matter, particularly if it is very small, in which case a better term to describe the relation would be the road *leads* to the park. According to Talmy (1983), at the fine-structural level of conceptual organization, language shows greater affinity with topology than with metric spaces; however, metric details occasionally overwrite topological properties, particularly in situations where small metric modifications imply topological changes. The importance of metric properties in people's mental maps is well-known (Cadwallader 1976; Cadwallader 1979; McNamara *et al*. 1994; McNamara and LeSueur 1989; Montello 1991; Montello 1992). Computational models that consider approximate distances either transform them into a Cartesian coordinate system to perform inferences (Davis 1986), or choose separate models for distances and directions (Frank 1992; Hernández *et al.* 1995; Hong *et al.* 1995) or distances and topological relations (Hernández 1994). Following the premise that *topology matters*, *metric refines* (Egenhofer and Mark 1995b), we develop here a two-tier model for the analysis of natural-language spatial relations:

(1) Capturing the topology of the configuration, and

(2) analyzing the topological configuration according to a set of metric properties.

This approach is different from related investigations into the semantics of spatial relations and spatial predicates. Work on how humans conceptualize spatial relations (Grimaud 1988; Japkowicz and Wiebe 1991) is fundamental to the study of human cognition of spatial relations; however, we do not explicitly use these ideas of conceptualization in our work, because our formalism is concerned with semantics that can be captured from the geometric configuration of spatial relations. Our focus is on the geometry of configurations and not the conceptualization of the spatial configuration. Positive results obtained from earlier work with human subject testing (Mark and Egenhofer 1994b) justify our assumption for taking this approach. Our study also relates to Rosch's (1973; 1978) general theory of human categorization whereby prototypical cases are used and objects are specified in terms of their distances from these prototypes. We are however dealing with spatial relations and not objects. In this respect, we are building on Herskovits's (1986) work, which used Rosch's method of prototypical categorization and applied it to spatial relations. Our goal is similar to that of the Visual Translator VITRA, which aims at translating from a visual into a linguistic mode (Herzog and Wazinski 1994). VITRA's computation algorithms for the basic meanings of topological relations use the distance between the located and reference objects, while for the computation of projective relations, the deviation angle of the located object from the canonical direction implied by the relation is also used. Our approach differs as it uses metric determination only as a secondary measure to refine the primary measure of topology.

The scope of this paper are binary relations between spatial objects that people conceptualize as a line and a region. Examples are the path of a hurricane with respect to a continent and a road's relation to a park. The critical components for line-region relations are the regions' interiors, boundaries, and exteriors and the lines' interiors and boundaries. When a region's interior, boundary, or exterior interacts with either the boundary or interior of a line, certain metric properties can be captured about this interaction. For instance, the interior of a line can share parts with the boundary of a region, and one could measure the length of the common stretch. A purely quantitative measure would record an absolute value, for instance the length of the common boundary in inches. Such an approach would be insufficient as it does not take into consideration the relation to the objects to which it belongs; therefore, under such operations as scaling of the entire scene, a different value would be obtained and stronger values would be obtained when a smaller reference object was chosen. Following Talmy's (1983) observation that the objects' sizes are irrelavant for the choice of their spatial relation term, we design a model for metric concepts that normalizes metric values for line-region relations with respect to the region's area, the line's length, and the region's perimeter. To describe details about topological relations, we consider two metric concepts:

(1) the *splitting*, which determines how the region's and line's interiors, boundaries, and exteriors are cut; and

(2) the *closeness*, which determines how far apart are the region's boundary and the parts of the line.

Alternative models for spatial relations have been proposed, most notably symbolic projections (Chang and Jungert 1996) and the region-connection calculus (Randell *et al.* 1992). *Symbolic projections* model spatial relations based on directions captured independently along the coordinate axes. Unlike the 9-intersection, however, they refer to the objects' minimum bounding rectangles, rather than to their actual shapes, which provides an approximation that depends on the objects' orientations. The *region-connection calculus*, based on the part-whole theory of *mereology* (Simons 1987) and Clarke's (1981) calculus of individuals, identifies for region-region configurations the same set of binary relations as the 9-intersection; however, it has not been developed for relations involving line-like objects.

The remainder of this paper presents the topological and metric models used to specify the geometry of spatial relations and demonstrates how this analysis helps to select appropriate natural-language terms. Section 2 summarizes the topological measures for line-region relations. Section 3 and 4 respectively introduce splitting ratios and closeness measures, the two classes of metric measures for spatial relations. Section 5 demonstrates how the topological and metric measures are used in spatial query processing as well as in generating a natural-language term to describe a spatial configuration. Section 6 closes with conclusions.

2 Measures for Topology

The 9-intersection is a comprehensive model for binary topological spatial relations and applies to objects of type area, line, and point (Egenhofer and Herring 1991). It characterizes the topological relation t between two point sets, A and B, by the set intersections of A's interior (A°), boundary (∂A), and exterior (A^-) with the interior, boundary, and exterior of B, called the *9-intersection* (Eq. 1).

$$I(A,B) = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix} \qquad (1)$$

With each of these nine intersections being empty ( ) or non-empty (¬), the model has 512 possible topological relations between two point sets, some of which cannot be realized, depending on the dimensions of the objects, and the dimensions of their embedding space (Egenhofer and Herring 1991). For two simple regions without holes embedded in R^2, the categorization shows eight distinct topological relations. Interior, boundary, and exterior of a line are defined according to algebraic topology (Spanier 1966): the boundary of a simple line comprises the two end points, the interior is the closure of the line minus the boundary, and the exterior is the complement of the closure. For two simple lines (non-branching, no self-intersections) embedded in R^2, 33 different topological relations can be realized with the 9-intersection, and for a line and a region, 19 different situations are found (Figure 1), which are the focus of this paper. More detailed distinctions are possible if further criteria are employed to evaluate the non-empty intersections.

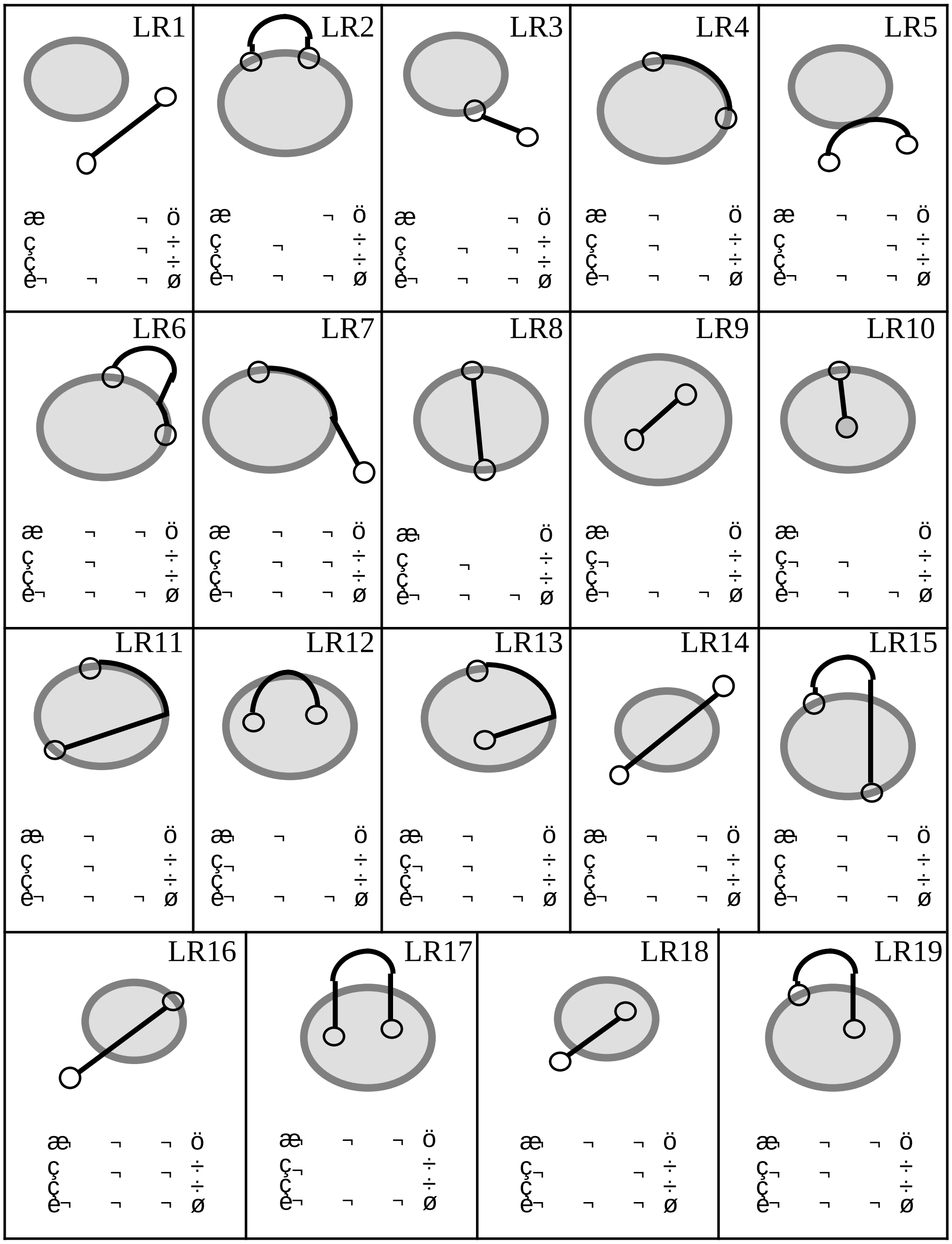


Figure 1: Geometric interpretations of the 19 line-region relations that can be realized from the 9-intersection (Egenhofer and Herring 1991).

The only other topological invariant used here is the concept of the *number of component.* A component is a separation of any of the nine intersections (Egenhofer and Franzosa 1995). The number of components of an intersection is denoted by $\#(A \cap B)$. For example, for line-region relation LR 14, $\#(L^\circ \cap \partial R) \geq 2$, whereas for LR 10, $\#(\partial L \cap \partial R) = 1$.

The 19 line-region relations can be arranged according to their topological neighborhoods (Egenhofer and Mark 1995a) based on the knowledge of the deformations that may change a topological relation by pulling or pushing the line's boundary or interior (Figure 2). The topological neighborhoods establish similarities that were shown to correspond to groupings people frequently make when using a particular natural-language term (Mark and Egenhofer 1994b). For example, the term *crosses* was found to correspond to the five relations located in the diagonal from the lower left to the upper right of the conceptual neighborhood diagram (LR 8 to LR 14 in Figure 2). Such groupings of the 9-intersection relations in the conceptual neighborhood diagram may serve as a high-level measure to define the meaning of natural-language spatial relations. However, topology *per se* may be insufficient as the only measure, particularly in border-line cases where small metric changes have a significant influence on topology.

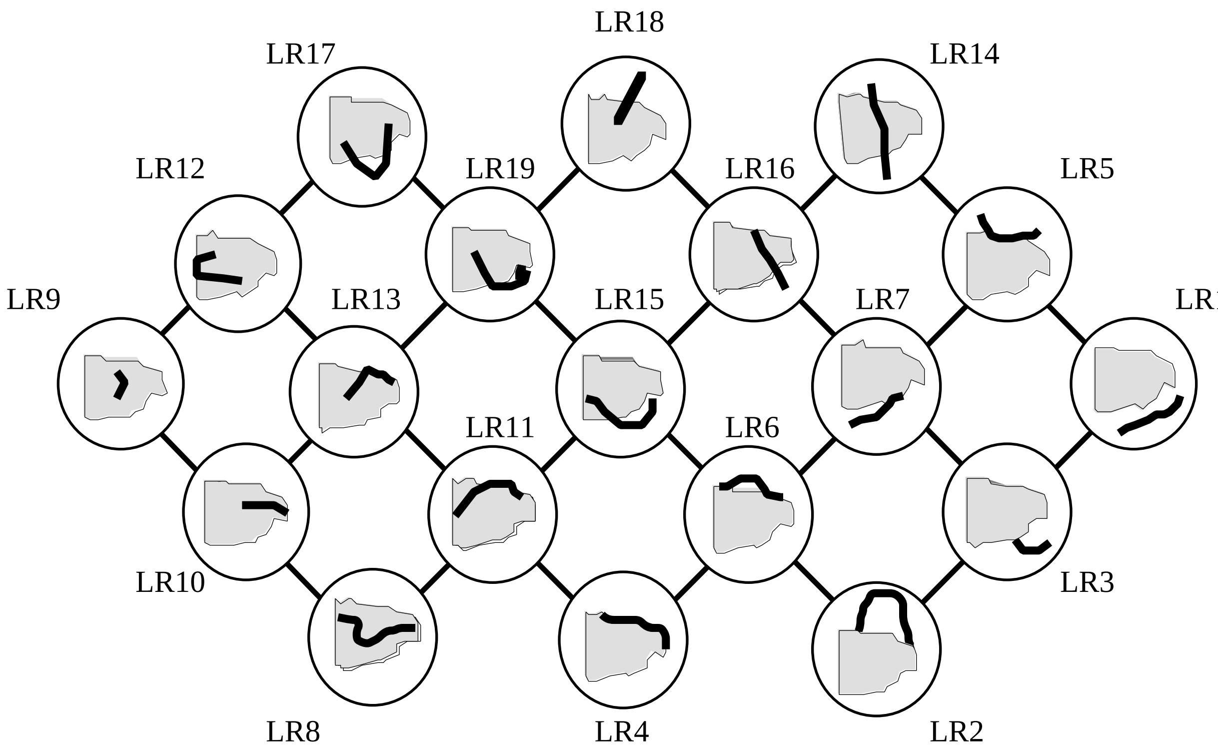


Figure 2: The conceptual neighborhood graph of the nineteen line-region relations (Egenhofer and Mark 1995a).

The following sections define two metric concepts—splitting and nearness—that apply to topological relations and may enhance each of the nineteen topological relations to distinguish more details.

3 Splitting

Splitting determines how a region's interior, boundary, and exterior are divided by a line's interior and boundary, and vice versa. To describe the degree of a splitting, the metric concepts of the length of a line and the area of a region are used. In the context of topological relations between lines and regions, *length* applies to the line's interior, any non-empty intersection with a line's interior, or their components; and to region boundaries, any non-empty intersection between a region's boundary and a line's exterior, or their components. *Area* applies to the interior or regions, the intersections between a line's exterior and a region's interior or exterior, and their components. Among the entries of the 9-intersection for a line and a region, there are seven intersection that can be evaluated with a length or an area (Table 1). Only the three intersections between the line's boundary cannot be evaluated with a length or area measure, because these intersections are 0-dimensional (i.e., points).

	R°	∂R	R^-
L°	$length(L^\circ \cap R^\circ)$	$length(L^\circ \cap \partial R)$	$length(L^\circ \cap R^-)$
∂L	—	—	—
L^-	$area(L^- \cap R^\circ)$	$length(L^- \cap \partial R)$	$area(L^- \cap R^-)$

Table 1: Area and length measures applied to the nine intersections of the line's interior (L°), boundary (∂L), and exterior (L^-) with the region's interior (R°), boundary (∂R), and exterior (R^-).

To normalize these lengths and areas, each of them is put into perspective with the line and the region: The two area intersections are compared with the area of the region, resulting in two splitting measures. Another ten splitting measures are obtained by comparing the four length intersections with the length of the line, and the length of the region's perimeter.

3.1 Inner Area Splitting

Inner area splitting describes how the line's interior divides the region's interior. With this separation a one-dimensional object splits a two-dimensional object into two (or more) parts such that parts of the region's interior are on one side of the line, and others are located on the opposite side of the line (Figure 3). Inner area splitting only applies to a subset of the 19 region-line relations. Those relations for which the line's interior intersects with the region's interior ($L^\circ \cap R^\circ = \neg$), but the line's boundaries is outside of the region's interior ($\partial L \cap R^\circ =$), always have a value for inner area splitting. In addition, inner area splitting may apply if the line's interior intersects with the region's boundary and interior ($L^\circ \cap R^\circ = \neg$ *and* $L^\circ \cap \partial R = \neg$) and the line's boundary intersects with the region's interior ($\partial L \cap R^\circ = \neg$). In such situations it is necessary that there are more components in the interior-interior intersection than there are components of the intersection between the line's boundary and the region's interior, i.e., $\#(L^\circ \cap R^\circ) > \#(\partial L \cap R^\circ)$.

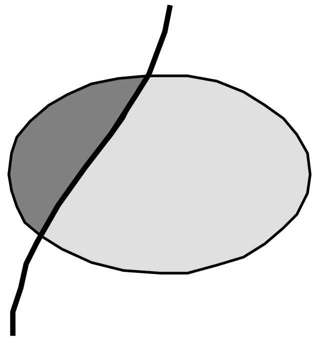


Figure 3: Inner area splitting: the line's interior divides the region's interior into parts on two opposite sides (more complex configurations may have multiple separations on either side of the line).

A normalized measure of this property is the *inner areasplitting ratio* (*IAS*) as the smaller sum of the areas on either side of the line—left and right are chosen arbitrarily and their choice does not influence the measure—over the total area of the region (Eq. 2). The range of *IAS* is $0 < IAS \leq 0.5$. It would reach 0 if the interior-interior intersection between the line and the region was empty, and is 0.5 if the line separates the region's interior into areas that total the same size on the left-hand side and the right-hand size.

$$IAS = \frac{\min(area(leftComponents(L^{-} \cap R^{\circ})), area(rightComponents(L^{-} \cap R^{\circ})))}{area(R)} \qquad (2)$$

3.2 Outer Area Splitting

Outer area splitting occurs if the line's interior interacts with the exterior of the region such that it produces separations of the exterior between the interior of the line and the boundary of the region. This involves a one-dimensional object that splits a two-dimensional object (the region's exterior) into two (or more) two-dimensional parts: (1) parts of the region's exterior that are bounded because they are completely surrounded by the line's interior and the region's boundary, and (2) parts of the region's exterior that are unbounded (Figure 4).

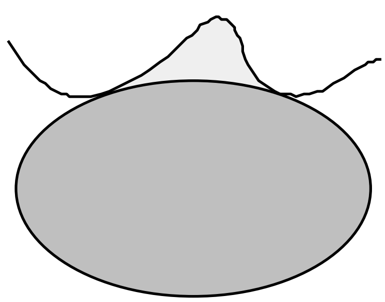


Figure 4: Outer area splitting: the line's interior divides the region's exterior into bounded and unbounded areas (more complex configurations may have multiple areas that are bounded by the same line).

Outer area splitting requires that the line's interior intersects with the region's exterior ($L^{\circ} \cap R^{-} = \neg$) and that the line's boundary is located in the region ($\partial L \cap R^{-} =$). Outer area splitting also may apply to configurations for which line interiors intersect with both the region's interior and boundary ($L^{\circ} \cap \partial R = \neg$ *and* $L^{\circ} \cap R^{-} = \neg$) and whose line boundaries intersect with the region's exterior ($\partial L \cap R^{-} = \neg$). For these situations, it is necessary that the region's exterior contains more components of the line's interior than of the line's boundary ($\#(L^{\circ} \cap R^{-}) > \#(\partial L \cap R^{-})$). A normalized measure of outer area splitting is the *outer area splitting ratio* (*OAS*) as the ratio of the sum of the region's area and the bounded exterior, which is the part of the exterior that is enclosed by the line's interior and the region's boundary, over the region's area (Eq. 3). It is greater than zero such that the larger the bounded area, the larger the splitting ratio. It would reach 0 if the bounded area was non-existent (i.e., either an empty intersection between the line's interior and the region's exterior, or an insufficient number of components in the intersection between the line's interior and the region's exterior.

$$OAS = \frac{area(boundedComponents(L^{-} \cap R^{-})}{area(R)} \tag{3}$$

3.3 Inner Traversal Splitting

The region's interior separates the line's interior into inner and outer line segments. This involves a two-dimensional object splitting a one-dimensional object into two one-dimensional parts (or sets of parts): line parts that are inside the closure of the region, and line parts outside of the region (Figure 5).

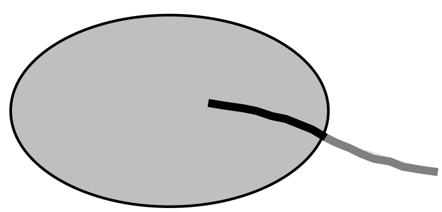


Figure 5: Inner traversal splitting: the region's interior divides the line into parts of inner and outer segments (more complex configurations may have multiple inner and outer segments for a line).

Inner traversal splitting applies to relations in which the line's interior is located at least partially in the region's interior ($L^{\circ} \cap R^{\circ} = \neg$). A normalized measure for the traversal is the *inner traversal splitting ratio* (*ITS*) between the length of the inner parts of the line and the length of the total line (Eq. 4). Its range is $0 < ITS \leq 1$. *ITS* would be 0 if the interior-interior intersection between the line and the region was empty. The greatest value is reached if the line's interior is completely contained in the region's interior.

$$ITS = \frac{length(L^{\circ} \cap R^{\circ})}{length(L)} \tag{4}$$

3.4 Entrance Splitting

While the inner traversal splitting normalizes the common interiors with respect to the line's length, the entrance splitting compares the length of the common interiors to the length of the region's boundary. It applies under the same conditions as the inner traversal splitting. Its measure, called the *entrance splitting ratio* (ENS), captures how far the line enters into the region (Eq. 5). All values of the entrance splitting ratio are greater than zero, but no upper bound exists.

$$ENS = \frac{length(L^\circ \cap R^\circ)}{length(\partial R)} \tag{5}$$

3.5 Outer Traversal Splitting

While the inner traversal splitting describes how much of the line is in the region's interior, the outer traversal splitting refers to the part of the line that is in the region's exterior. Outer traversal splitting applies to relations in which the line's interior is located at least partially in the regions' exterior ($L^\circ \cap R^- = \neg$). A normalized measure for the traversal is the *outer traversal splitting ratio* (OTS) between the length of the outer parts of the line and the length of the total line (Eq. 6).

$$OTS = \frac{length(L^\circ \cap R^-)}{length(L)} \tag{6}$$

3.6 Exit Splitting

Analog to the pair of inner traversal splitting and entrance splitting, the outer traversal splitting has a dual, the *exit splitting.* It captures how far the line exits the region, and applies under the same conditions as the outer traversal splitting. The *exit splitting ratio* (EXS) normalizes the length of the line's interior that lays in the region's exterior with respect to the length of the region's boundary (Eq. 7). It is greater than 0 and has no upper bound.

$$EXS = \frac{length(L^\circ \cap R^-)}{length(\partial R)} \tag{7}$$

3.7 Line Alongness

The region's boundary interacts with the line's interior such that it separates the line into two sets of line parts: line segments that are outside of the region's boundary (i.e., either in the region's interior or exterior), and line segments that are contained in the boundary. This separation makes a one-dimensional object splitting another one-dimensional object into two or more one-dimensional parts (Figure 6).

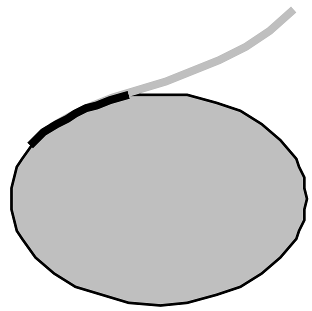


Figure 6: Line alongness: the region's boundary separates the line's interior into parts of outer and inner segments (more complex configurations may have multiple components in the intersection between the region's boundary and the line's interior).

In order to consider line alongness, the line's interior must intersect with the region's boundary ($L^{\circ} \cap \partial R = \neg$). As the measure for the separation, we introduce the notion and concept *line alongness ratio* (LA) as the ratio between the length of all line parts contained in the boundary, and the total length of the line (Eq. 8). The range of the line alongness ratio is $0 \leq LA \leq 1$. LA is 0 if the line intersects the region's boundary exclusively in 0-dimensional components, and it reaches 1 if $L^{\circ} \quad \partial R$.

$$LA = \frac{length(L^{\circ} \cap \partial R)}{length(L)} \tag{8}$$

3.8 Perimeter Alongness

The line's interior separates the region's boundary into two sets of objects, one that coincides whit the line's interior, and another that is disjoint from the line's interior. The separation is such that a one-dimensional object splits another one-dimensional object into two (or more) one-dimensional objects. The perimeter alongness can be measured for relations in which the line's interior intersects with the region's boundary ($L^{\circ} \cap \partial R = \neg$). The perimeter alongness is measured by the ratio between the length of coinciding parts between the line's interior and the region's boundary and the perimeter, called the *perimeter alongness ratio* (PA) (Eq. 9). The range of the perimeter alongness ratio is $0 \leq PA < 1$. PA is 0 if the interior-boundary intersection between the line and the region consists exclusively of disconnected 0-dimensional components. PA would reach the maximum of 1 if cycles were permitted as lines and such a cycle would coincide with the region's boundary.

$$PA = \frac{length(L^{\circ} \cap \partial R)}{length(\partial R)} \tag{9}$$

3.9 Perimeter Splitting

Perimeter splitting occurs if the line splits the region's boundary into two or more parts. This involves two (or more) zero-dimensional or one-dimensional objects—the line's boundary or interior—cutting another one-dimensional object (the region's boundary) (Figure 7).

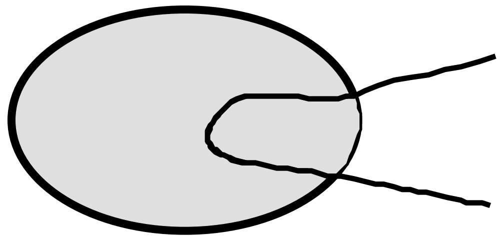


Figure 7: Perimeter splitting: the line separates the region's boundary into segments (more complex configurations may create multiple segments in the region's boundary).

Perimeter splitting requires that the line intersects the region's boundary ($L^\circ \cap \partial R = \neg$ *or* $\partial L \cap \partial R = \neg$) such that the region's boundary is split into at least two components ($\#(\partial R - L) \geq 2$). The perimeter splitting ratio (PS) is the ratio between the longest of these components and the region's perimeter (Eq. 10). Its range is $0 < PS < 1$.

$$PS = \frac{\max(length(components(L^{-} \cap \partial R)))}{length(\partial R)} \qquad (10)$$

3.10 Length Splitting

While the perimeter splitting compares the length of the longest perimeter component with the total length of the perimeter, the length splitting compares it with the length of the line. The metric measure is the *line splitting ratio* (LS) (Eq. 11), which is great than 0 without an upper bound.

$$LS = \frac{\max(length(components(L^{-} \cap \partial R)))}{length(L)} \qquad (11)$$

3.11 Comparison of the Splitting Ratios

Each splitting ratio applies to several different topological relations. Figure 8 shows how the criteria for the ten splitting ratios map onto the conceptual neighborhood graph of the line-region relations (Egenhofer *et al.* 1993). Each constraint covers a contiguous area.

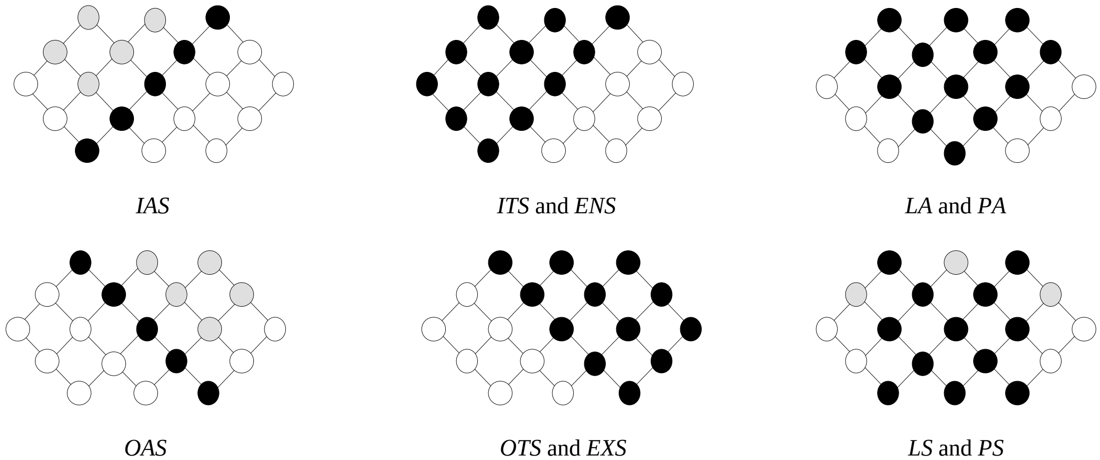


Figure 8: The relations that qualify for inner area splitting (IAS), outer area splitting (OAS), inner traversal splitting (ITS), entrance splitting (ENS), outer traversal splitting (OTS), exit splitting (EXS), line alongness (LA), perimeter alongness (PA), line splitting (LS), and perimeter splitting (PS). Black, gray, and white indicate that the metric measure applies always, sometimes, and never, respectively.

4 Closeness

Unlike splitting, which requires coincidence and describes how much is in common between two objects, *closeness* describes how far apart disjoint parts are. The object parts involved are the boundary and the interior of the line, and the boundary of the region. There is no need to consider the region's interior, since it is delineated by its boundary, and therefore no additional information in R^2 could be found by considering it in addition to the region's boundary.

Closeness involves considerations of distances among points and lines. For the configurations considered, there are four types of closeness measures of interest (the metric axioms for distances apply, i.e., there is a null element, distances are symmetric, and the triangle inequality holds):

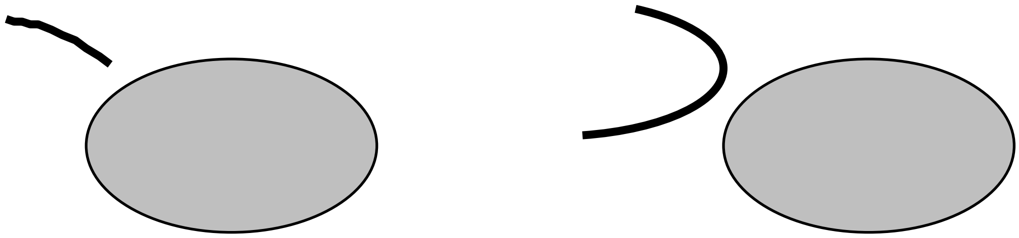
(1) the distance between a line's boundary and the region's boundary if the line's boundary is located in the exterior of the region;

(2) the distance between a line's boundary and the region's boundary if the line's boundary is located in the interior of the region;

(3) the distance of the shortest path between a line's interior and the region's boundary if the line's interior is located in the exterior of the region; and

(4) the distance of the shortest path between a line's interior and the region's boundary if the line's interior is located in the interior of the region.

The closeness measures are not completely orthogonal, since depending on the shape of the line or the region, they may have the same values. For instance, for the configuration in Figure 9a, the distance from the region's boundary to the line's boundary (i.e., its two endpoints as defined in Section 2) is the same as the distance from the region's boundary to the line's interior, since the line's boundary is the line's closest part to the region's boundary; however, in Figure 9b, the same parameters have different values because the line's interior is closer to the region's boundary than the line's boundary.



(a) (b)

Figure 9: Two configurations with (a) identical and (b) different values for the distance measures from the line's boundary and interior to the region's boundary.

Distances are commonly defined between points; however, the closeness measures require distance measures between a point and a line, or between two lines.

Definition 1: The distance between a point p and the boundary of a region (∂R) is defined as the length of the shortest path from p to ∂R (Eq. 12).

$$\begin{aligned} dist(p,\partial R) &= dist(p, r \quad \partial R) \\ &/ \quad q \quad \partial R \quad | \quad dist(p,q) < dist(p,r) \end{aligned} \tag{12}$$

Therefore, there is no other point on the region's boundary that would be closer to p

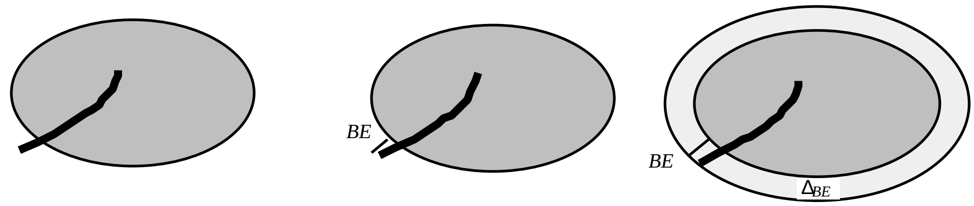
Definition 2: The distance between the interior of a line (L°) and the boundary of a region (∂R) is defined as the length of the shortest path from L to ∂R (Eq. 13).

$$\begin{aligned} dist(L^\circ,\partial R) &= dist(l \quad L^\circ, r \quad \partial R) \\ &/ \quad (p \quad L^\circ, q \quad \partial R) \quad | \quad dist(p,q) < d(l,r) \end{aligned} \tag{13}$$

Therefore, there are no other parts in the line's interior that would be closer to any point on the region's boundary

4.1 Outer Closeness

The *outer closeness* describes the remoteness of the region's boundary ∂R from p, a boundary point of a line located in the exterior of the region (Figure 10a). Outer closeness only applies to those line-region relations with at least one point of the line's boundary being located in the region's exterior ($\partial L \cap R \quad = \neg$). A purely quantitative measure for the remoteness would be the distance between the region's boundary and the line's boundary point(s) in the region's exterior (Figure 10b). It is the shortest connections between the line's boundary and the region, i.e., there exists no other point in the region's boundary that would be closer to the line's boundary (Eq. 14). Since this measure is only applicable if $\partial L \cap R^- = \neg$, it can never be 0.



(a) (b) (c)

Figure 10: Outer closeness: (a) the line's boundary in the region's exterior, (b) the remoteness measure BE from the region's boundary to the line's boundary, and (c) the region's outer buffer zone as an equi-distant enlargement of the region.

$$BE = \min(dist\,(p, \partial R)) \quad | \quad p \quad (\partial L \cap R^{-}) \tag{14}$$

While the actual distance between the two boundaries is a precise measure, it varies significantly with the scale of the representation. For instance, a scaling by a factor of 2 would make any two objects be twice as much remote. A variety of dimension-independent measures could be thought of, such as the proportion by which the line would have to be extended, or shrunken, so that its boundary coincides with the region's boundary. We selected two outer closeness measures: (1) the outer line closeness as the ratio between the distance from the line's boundary to the region's boundary, and the line's length (Figure 10b), and the outer area closeness as the ratio between the area made up by an equi-distant enlargement of the region—also known as a *buffer zone* (Laurini and Thompson 1992)—and the actual area (Figure 10c).

We define the *outer area closeness* measure (*OAC*) in terms of the area of the region R and the area made up by the buffer zone, denoted by $\Delta_{BE}(R)$. It is of width BE and extends into the region's exterior (Eq. 15). *OAC* is greater than 0 with no upper bound, and would be 0 if BE were 0. The normalization $area(\Delta_{BE}(R))/(area\,(\Delta_{BE}(R))+area\,(R))$ would produce values between 0 and 1, however, the distribution would be non linear, particularly for $area(R) \ll area(\Delta_{BE}(R))$.

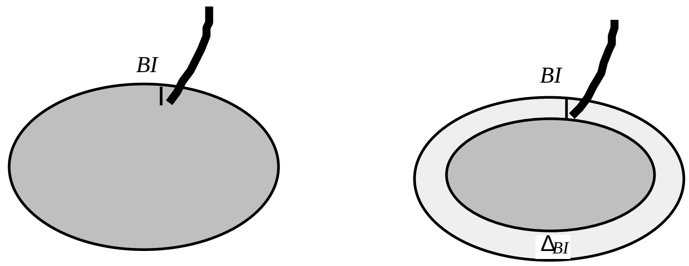
$$OAC = \frac{area(\Delta_{BE}(R))}{area(R)} \tag{15}$$

The *outer line closeness* measure (*OLC*) is defined in terms of BE, the distance from the line's boundary to the region's boundary, and the line's length (Eq. 16). Its values are greater than 0 without an upper bound. It would be 0 if BE were 0.

$$OLC = \frac{BE}{length(L)} \tag{16}$$

4.2 Inner Closeness

Analogous to the outer closeness, the *inner closeness* captures the remoteness of the line's boundary, located in the interior of the region (criterion: $\partial L \cap R^{\circ} = \neg$), from the region's boundary (Figure 11a). The mere distance between the boundaries of the region and the line are captured by a quantitative measure BI (Eq. 17). This distance is greater than 0, because the line's boundary must be located in the region's interior. If both boundary points of the line are inside R, then BI is the distance of that boundary point closest to the region's boundary.



(a) (b)

Figure 11: Inner closeness: (a) the line's boundary in the region's interior and (b) the region's inner buffer zone as an equi-distant reduction of the region.

$$BI = \min(dist\,(p, \partial R)) \quad | \quad p \quad (\partial L \cap R^{\circ}) \tag{17}$$

The *inner area closeness* (IAC) is then defined as the ratio between the area made up by an equi-distant reduction of the region and the actual area (Figure 11b). The buffer zone $\Delta_{BI}(R)$ has the width b and is taken from the region's boundary into the region's interior (Eq. 18). Its rage is $0 < IAC < 1$.

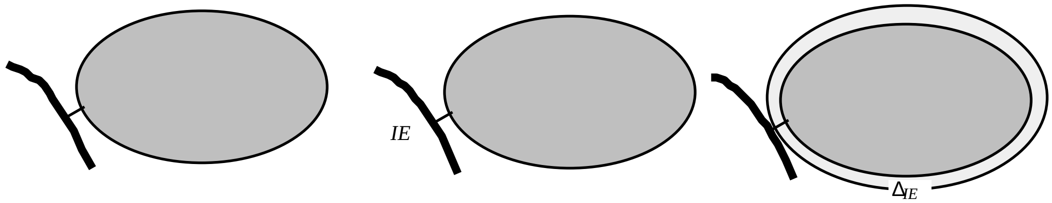
$$IAC = \frac{\Delta_{BI}(R)}{area(R)} \tag{18}$$

The *inner line closeness* (ILC) refers to the relative amount the line has to be extended or shortened to coincide with the region's boundary. The increment is normalized with respect to the line's actual length (Eq. 19).

$$ILC = \frac{BI}{length(L)} \tag{19}$$

4.3 Outer Nearness

The outer nearness describes how far the line's interior is from the region's boundary (Figure 12a). It only applies to one line-region relation, namely the one with the line's boundary and interior completely contained in the region's exterior ($L \quad R^{-}$). The quantitative measure for outer nearness is the length of the shortest connection between the line and the region (Figure 12b). It is always greater than zero, because L must be completely contained in R's exterior (Eq. 20).



(a) (b) (c)

Figure 12: Outer nearness: (a) the line is completely contained in the region's exterior, (b) the remoteness measure *IE* from the region's boundary to the line's interior, and (c) the region's outer buffer zone as an equi-distant enlargement of the region.

$$IE = dist(L^\circ, \partial R) \quad | \quad L \quad R^- \tag{20}$$

The *outer area nearness* (*OAN*) is then defined as the ratio between the area made up of an equi-distant reduction of the region of width *IE*, denoted by $\Delta_{II}(R)$, and the actual area of the region R (Eq. 21). *OAN*'s values are greater than 0, with no upper bound. *OAN* would be 0 if *IE* were 0.

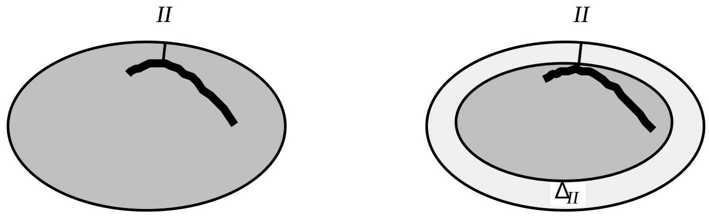
$$OAN = \frac{area(\Delta_{IE}(R))}{area(R)} \tag{21}$$

The *outer line nearness* (*OLN*) normalizes the length by which the line would have to be extended or shortened such that its boundary would coincide with the region's boundary, with respect to the length of the initial line (Eq. 22). The values of the outer line nearness are greater than 0 and increase linearly with the length of *IE*.

$$OLN = \frac{IE}{length(L)} \tag{22}$$

4.4 Inner Nearness

Complementary to the outer nearness, the inner nearness describes how far the line's interior, located in the interior of the region (criterion: $L \quad R^\circ$), is from the region's boundary (Figure 13a) This distance is greater than zero, because the line must be completely contained in the region's interior (Eq. 23).



(a) (b)

Figure 13: Inner nearness: (a) the line completely contained in the region's interior and (b) the region's inner buffer zone as an equi-distance reduction of the region.

$$II = dist(L^\circ, \partial R) \quad | \quad L \quad R^\circ \tag{23}$$

The *inner area nearness* (IAN) is then defined as the ratio between the area made up by a buffer zone of width II, denoted by $\Delta_{II}(R)$, that extends from the boundary into the region's interior (Figure 13b). Its range is $0 < IAN < 1$ (Eq. 24).

$$IAN = \frac{area(\Delta_{II}(R))}{area(R)} \tag{24}$$

The *inner line nearness* (ILN) captures by how much the line would have to be extended in order to intersect with the region's boundary. It is measured as the ratio between the distance to the region's boundary and the length of the line (Eq. 25). The values of the inner line nearness must be greater than zero.

$$ILN = \frac{II}{length(L)} \tag{25}$$

4.5 Comparison of the Closeness Measures

From the criteria for the closeness measures, one can derive which topological relations may be refined by the corresponding measures (Figure 14). Except for the six topological relations in the bottom triangle of the neighborhood graph, all relations have at least one closeness measure. Those six relations without a closeness measure are such that both line boundaries coincide with the region's boundary, therefore, the distances from the line's parts to the boundaries region are all zero and no refinements can be made to these relations.

5 Parsing and Translating a Graphical Relation into a Verbal Expression

With the two sets of parameters we can perform a detailed analysis of a simple spatial configuration with a line and a region, capturing the configuration's topology and analyzing it according to its metric properties. This *per se* would provide the basis for a computational comparison of two or more spatial configurations for similarity (Bruns and Egenhofer 1996). Here we pursue a different path by mapping the parsed configuration onto a natural-language term that would best describe the spatial relation between the two geometric objects. For the time being, any semantic or presentational aspects (Mark *et al.* 1995) are ignored in this mapping.

The mappings from the topological and metric measures onto corresponding natural-language terms are based on results from human-subject experiments (Shariff 1996). A total of sixty-four English-language terms were tested, for which subjects sketched a road with respect to a given outline of a park such that the sketch would match the corresponding natural-language term that describes the spatial relation. By analyzing the sketches' topological relations and their splitting and closeness measures, we obtained the mappings from the geometry of a configuration onto the corresponding, significant parameters and their values. Significant parameters were distinguished from non-significant ones through a cluster analysis (Shariff 1996). The criterion for a parameter to be considered significant for a specific spatial term was that its standard score was greater than one (i.e., the mean of such a parameter is at least one standard deviation higher than the mean of the entire data set). To demonstrate how the model developed here can be used for such translations, we give three examples in which the spatial relation of a geometric configuration is translated into a natural-language spatial term.

5.1 Example 1

Figure 15 shows a configuration in which a line (e.g., a road) crosses the boundary of a region (e.g., a park). Based on the topology (LR 18), the applicable metric parameters for splitting and closeness are found in Figures 8 and 14, respectively.

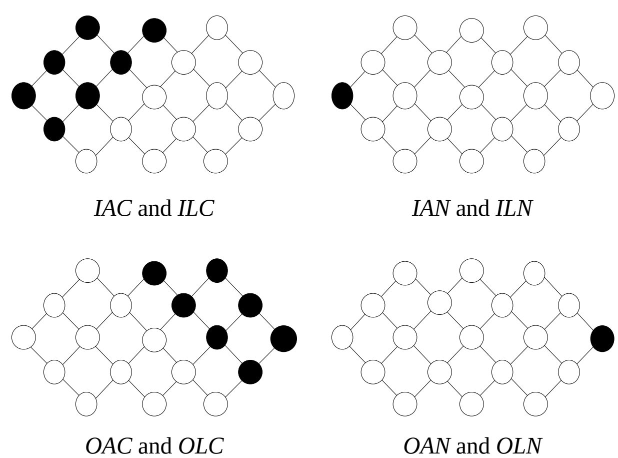


Figure 14: The relations that qualify for inner area closeness (IAC), inner line closeness (ILC), outer area closeness (OAC), outer line closeness (OLC), inner area nearness (IAN), inner line nearness (ILN), outer area nearness (OAN), and outer line nearness (OLN).

The human-subject tests found that only a subset of these parameters—inner traversal splitting, outer traversal splitting, inner area closeness, and outer area closeness—are significant for the terms that are represented by LR 18. Table 2 shows a sample of eight terms—*ends at*, *ends in*, *ends just inside*, *ends outside*, *enters*, *goes into*, *goes out*, and *goes to*—that apply to LR 18, together with the significant parameters. For each parameter, the mean value (i.e., the best fit) and the range of values is given. The *value range* of a metric parameter refers to the minimum and maximum value obtained from the subjects' sketches. The goal is now to determine which of these terms are a better match for the particular configuration, and which do not convey the meaning the meaning of the configuration.

Topological Relation	Spatial Term	ITS		ETS		IAC		OAC	
		mean	range	mean	range	mean	range	mean	range
LR 18	ends at	0.24	0.02–0.65	0.76	0.36–0.98	0.79	0.19–0.98	7.69	2.09–20.73
LR 18	ends in	0.51	0.17–0.91	0.49	0.09–0.83	0.81	0.18–0.99	3.66	0.59–9.94
LR 18	ends just inside	0.16	0.05–0.71	0.84	0.29–0.95	0.55	0.05–0.89	3.95	0.86–10.27
LR 18	ends outside	0.67	0.47–0.90	0.33	0.10–0.53	0.59	0.07–0.90	1.83	0.63–6.79
LR 18	enters	0.46	0.18–0.83	0.54	0.17–0.82	0.86	0.42–0.99	2.47	1.06–9.50
LR 18	goes into	0.40	0.18–0.76	0.60	0.24–0.82	0.81	0.29–0.99	3.78	1.25–9.29
LR 18	goes out	0.48	0.18–0.75	0.52	0.25–0.82	0.80	0.38–0.99	2.22	0.64–10.86
LR 18	goes to	0.20	0.04–0.57	0.81	0.44–0.96	0.75	0.18–0.99	7.37	1.63–12.04

Table 2: Spatial terms of topological relation LR 18, with means and value ranges of their significant parameters for splitting and closeness measures.

Table 3 summarizes for the four parameters how they are calculated and provides the values obtained for the configuration in Figure 15.

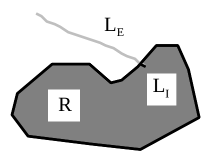
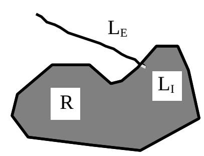
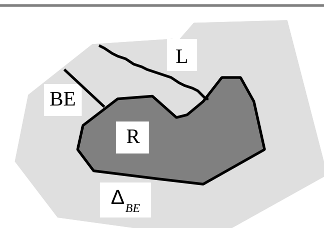
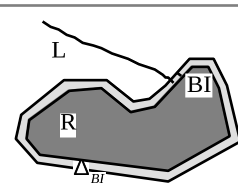
inner traversal splitting	$ITS = \dfrac{length(L_I)}{length(L)}$		$ITS = 0.07$
outer traversal splitting	$OTS = \dfrac{length(L_E)}{length(L)}$		$OTS = 0.93$
outer area closeness	$OAC = \dfrac{area(\Delta_{BE})}{area(R)}$		$OAC = 2.87$
inner area closeness	$IAC = \dfrac{area(\Delta_{BI})}{area(R)}$		$IAC = 0.28$

Table 3: Calculating the inner traversal splitting, the outer traversal splitting, the outer area closeness, and the inner area closeness for the configuration displayed in Figure 15.

By comparing these values with the calibrated model, the terms are ranked according to best fit. The terms *ends in*, *ends outside*, *enters*, *goes into*, and *goes out* fall outside of the value ranges of at least two parameters (Table 2) and, therefore, these terms are not considered for this configuration. Among the remaining three terms, *ends just inside* is the best fit for three parameters; *goes to* is the second best for three parameters, and *ends at* ranks third in three out of four times. Therefore, the sentence, "The road ends just inside the park" would be selected as the best fit, while valid alternatives would be, "The road goes to the park" or "The road ends at the park."

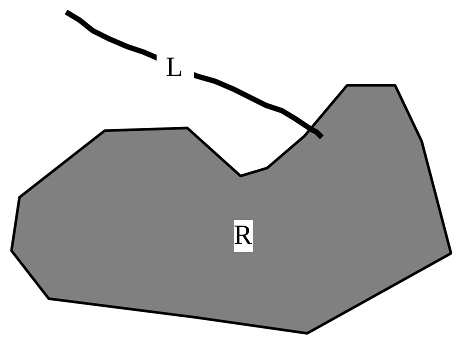


Figure 15: Does the line *enter* or *end just inside* the region?

5.2 Example 2

Figure 16 shows a configuration in which a line intersects a region such that it is close to the region's boundary from the inside and farther from the region's boundary in the exterior. A sample of terms that may fit this description are *crosses*, *cuts through*, *goes through*, *runs into*, and *splits.*

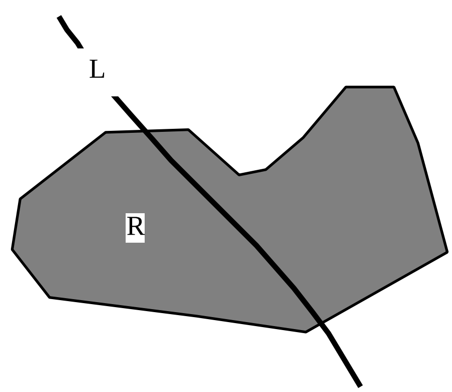


Figure 16: Does the line *cross* or *cut through* the region?

For the configuration's topological relation, LR 14, the human-subject tests found two metric parameters to be significant: inner area splitting and outer area closeness. Table 4 displays the mean and the value range for each parameter.

Topological	Spatial Term	IAS	OAC

Relation		mean	range	mean	range
LR 14	crosses	0.37	0.03–0.50	1.63	0.29–8.62
LR 14	cuts through	0.32	0.01–0.50	1.75	0.41–6.04
LR 14	goes through	0.40	0.03–0.50	2.17	0.70–8.21
LR 14	runs into	0.13	0.09–0.44	3.42	0.54–11.96
LR 14	splits	0.41	0.02–0.50	1.62	0.42–6.90

Table 4: Spatial terms of topological relation LR 14, with means and value ranges of their significant parameters for splitting and closeness measures.

For the configuration in Figure 16, the term *runs into* does not qualify, because the configuration is not located within the range of the inner area splitting. From among the remaining four spatial terms, *splits* comes closest to the mean values of inner area splitting and outer area closeness; therefore, it is selected as the term to describe the configuration. The ranking of the terms in between is more difficult, because they are subject to more subtle differences. Certainly, *crosses* would be better to describe the scene than *cuts through*, since both parameters have values that are closer to the mean of *crosses* than to the mean of cuts *through*. The term *goes through*, however, has a better match with the inner area closeness than both *crosses* and *goes through* have, however, it ranks considerably lower in the outer area closeness.

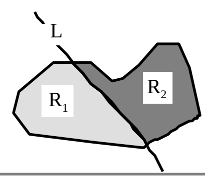
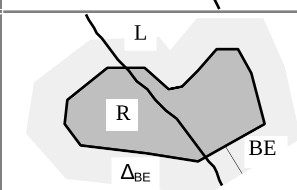
inner area splitting	$IAS = \dfrac{\min(area(R1), area(R2))}{area(R1) + area(R2)}$		$IAS = 0.49$
outer area closeness	$OAC = \dfrac{area(\Delta_{BE})}{area(R)}$		$OAC = 1.47$

Table 5: Calculating the inner area splitting and the outer area closeness for the configuration displayed in Figure 16.

5.3 Example 3

The following characteristics describe the configuration in Figure 17, in which a line is outside of the region, but follows the shape of the region. Candidate terms to describe this configuration are *bypasses*, *goes up to*, and *runs along* (Table 6).

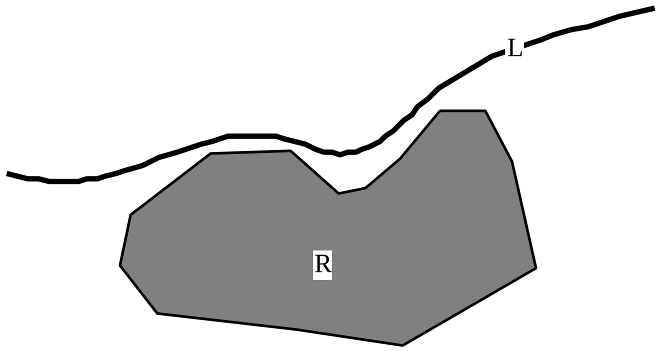


Figure 17: Does the line *run along* or *bypass* the region?

Topological Relation	Spatial Term	OAN		OAC	
		mean	range	mean	range
LR 1	bypasses	0.76	0.28–1.36	4.01	0.67–18.31
LR 1	goes up to	0.33	0.03–0.75	0.39	0.03–4.79
LR 1	runs along	0.33	0.16–1.29	1.06	0.30–7.13

Table 6: Spatial terms of topological relation LR 1, with means and value ranges of their significant parameters for splitting and closeness measures.

Based on the topological relation, LR 1, the significant parameters are outer area closeness and outer area nearness. The term *bypasses* does not fall within the ranges of outer area nearness or outer area closeness (Table 7), and is therefore not considered. Both terms *goes up to* and *runs along* have the same values for outer area nearness, but since *runs along* has a significantly lower value for the outer area closeness, it is chosen as the better term to describe the configuration than *goes up to.*

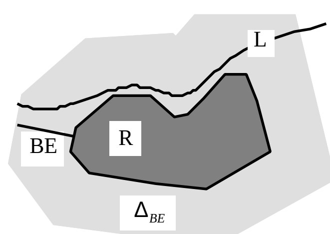
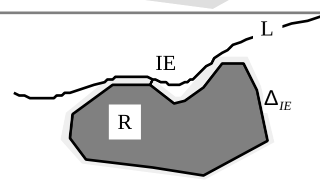
outer area closeness	$OAC = \frac{area(\Delta_{BE})}{area(R)}$		$OAC = 2.87$
outer area nearness	$OAN = \frac{area(\Delta_{IE})}{area(R)}$		$OAN = 0.24$

Table 7: Calculating the outer area closeness and the outer area nearness for the configuration displayed in Figure 17.

6 Conclusions

This paper developed a computational model to describe the semantics of natural-language spatial terms based on their geometry. The model is based on the 9-intersection topological model and refines is with metric details in the form of splitting and closeness ratios. Splitting ratios describe the proportion of an intersection with respect to the interior or boundary of the two objects. Their normalized values all fall within the interval between 0 and 1 and grow linearly with the size of the intersection. Closeness ratios specify distances between boundaries and interiors. For inclusion or containment relations, the (inner) closeness ratios are normalized to range between 0 and 1, while closeness ratios for disjoint relations are greater than zero with no upper limit. While this may appear to be an inconsistency in the model, it is necessary to obtain measures that grow linearly with the distance between the parts. The model was only developed for relations between a region and a line, however, the concepts generalize to relations between other geometric types, such as two regions or two lines.

Splitting and closeness measures can be implemented with standard GIS software. A prototype implementation with the Arc/Info GIS, however, requires the separation of the two objects into different layers (Shariff 1996). A method for computing the intersections necessary to determine the topological relation, using the "Identity" command, was described by Mark and Xia (1994). In order to determine the metric parameters, AMLs were written to compute intersections, lengths, and areas. Although this method demonstrated the feasibility of implementing the required operators with a commercial GIS, it was cumbersome, because Arc/Info does not support an object concept, and performance was slow. The use of GIS data structures that support an object model, and the integration of algorithms that are tailored to the operations necessary for efficient implementations of the 9-intersection and the metric refinements, are subjects for future investigations.

The model developed applies to a number of applications in the area of spatial reasoning, such as similarity retrieval and intelligent spatial query languages. We demonstrated how to use the model to generate natural-languages terms for simple spatial configurations. Based on a calibration of the 9-intersection with splitting ratios and closeness ratios, using human-subjects experiments for sixty-four English-language (Shariff 1996), we showed how a geometric configuration with a linear and an areal object can be analyzed to determine the pertinent features of their spatial relations. Values obtained from this method lead to the selection of appropriate natural-language spatial terms for such spatial scenes.

While the splitting and closeness ratios as refinements of topology cover much of the critical properties of the spatial relations, there are other parameters left that may make additional contributions to better choices of natural-language terms. Further investigations—both formalizations and human-subject tests—are necessary to develop a comprehensive and robust set of definitions of the semantics of natural-languages spatial relations. Some of these considerations were outlined in a larger-scale research plan (Mark *et al*., 1995). The most obvious aspect to study is the influence of the meanings of the objects on the choice of the spatial terms. Whether the objects or concern are roads and parks vs. hurricanes and islands, may lead to different mappings from topology and metric refinements onto the same spatial terms. With respect to geometry, the current model abstracts away all influences of orientation. This is a valid approach for modeling all those concepts and terms that are independent of orientation (such as those based primarily on containment, neighborhood, and closeness; however, orientation, is another parameter that may be critical for those relations expressing information about direction. For example, orientation may be important to distinguish *north* from *south* (or *above* from *underneath*) Orientations are invariant under translations and scaling, but they may change under rotation. The orientation of the objects can be assessed in several different ways: (1) the global cardinal relation between two objects, i.e., a relation with respect to a fixed orientation framework; (2) the orientation of an individual object, i.e., the cardinal relation between the object's major axis and a global reference frame; and (3) a local relation, i.e., the cardinal direction with respect to the framework established by one of the two objects' orientations. Similar to the metric properties, one could consider purely quantitative measures, e.g., in the form of degrees. Since people usually do not make such a fine distinction, coarser, qualitative models are necessary to formalize the properties of the three orientation concepts.

7 References

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