# A Granular Point Position Calculus

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## 1 Introduction

A qualitative representation provides mechanisms which characterize central essential properties of objects or configurations. A quantitative representation establishes a measure in relation to a general standard of measure which is generally usable.

The constant general availability of common measures is now self evident. However, one needs only remember the example of the history of technologies of measurement of length to see that the more local relative measures, which are qualitatively represented, (for example, "one piece of material is longer than another" versus "this thing is two meters long") can be managed by biological/epigenetic cognitive systems much more easily as absolute quantitative representations.

The two main trends in Qualitative Spatial Reasoning are topological reasoning about regions [3, 9] and positional reasoning about point configurations [4, 10]. Especially positional reasoning is important for robot navigation [8].

Typically, in Qualitative Spatial Reasoning relatively coarse distinctions between configurations are made only. Applications exist in which finer qualitative acceptance areas are helpful. The possibility to use finer qualitative distinctions can be viewed as a stepwise transition to quantitative knowledge. The idea of using context dependant direction and distance intervals for the representation of spatial knowledge can be traced back to Clementini, di Felice, and Hernandez [2]. However, only special cases of reasoning were considered in their work. Here, we will propose a calculus that makes direct use of general purpose constraint propagation. Thereby robot applications using safe spatial reasoning will be made possible.

## 2 Generalizing ternary point configuration calculi

The newly proposed calculus is called granular point configuration calculus GPCC. In this calculus two points are the basis for a reference system. The reference system can be interpreted as a partition of the plane into acceptance regions for a third point. All options for places of the third point which are in the same part of the partition are considered to be in an aquivalence class and are treated in the same way in categorization and reasoning tasks by subsequent modules. One variant of the GPCC calculus and its partition on the plane is shown in figure 1.

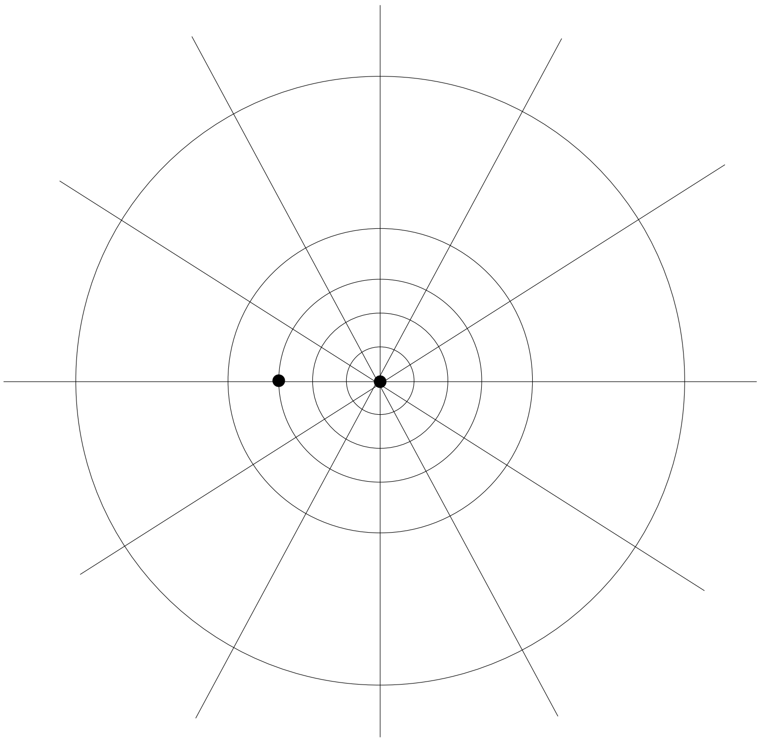

Figure 1: The partition of the $\text{GPCC}_3$-Calculus

To give a precise, geometric definition of the GPCC-relations we describe the corresponding geometric configurations in an analogue way to the TPCC calculus [6] on the basis of a Cartesian coordinate system represented by $\mathbb{R}^2$. First we define the special cases for $A = (x_A, y_A)$, $B = (x_B, y_B)$ and $C = (x_C, y_C)$.

$$A, B \text{ dou } C \quad := \quad x_A = x_B \wedge y_A = y_B \wedge (x_C \neq x_A \vee y_C \neq y_A)$$
$$A, B \text{ tri } C \quad := \quad x_A = x_B = x_C \wedge y_A = y_B = y_C$$

For the cases with $A \neq B$ we define a relative radius $r_{A,B,C}$

$$r_{A,B,C} \quad := \quad \frac{\sqrt{(x_C - x_B)^2 + (y_C - y_B)^2}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}$$

$$A, B \operatorname{sam} C \quad := \quad r_{A,B,C} = 0$$

and for $A \neq B \neq C$ a relative angle $\phi_{A,B,C}$:

$$\phi_{A,B,C} \quad := \quad \tan^{-1} \frac{y_C - y_B}{x_C - x_B} - \tan^{-1} \frac{y_B - y_A}{x_B - x_A}$$

The further base relations have an acceptance area depending on the granularity of the calculus to be applied. The calculus shown in figure 1, GPCC$_3$, has a level of granularity of 3 and 267 relations. A calculus of the granularity level $m$, described below as GPCC$_m$, has $(4m-1)(8m)+3$ base relations. The base relations of GPCC$_3$ are thus defined:

$$\begin{array}{rcl}
A, B \ {}_3\!\perp^1_0 \ C & := & 0 < r_{A,B,C} \leq 1/3 \wedge \phi_{A,B,C} = 0 \\
A, B \ {}_3\!\perp^1_1 \ C & := & 0 < r_{A,B,C} \leq 1/3 \wedge 0 \leq \phi_{A,B,C} \leq 1/6\pi \\
A, B \ {}_3\!\perp^1_2 \ C & := & 0 < r_{A,B,C} \leq 1/3 \wedge \phi_{A,B,C} = 1/6\pi \\
A, B \ {}_3\!\perp^1_3 \ C & := & 0 < r_{A,B,C} \leq 1/3 \wedge 1/6\pi \leq \phi_{A,B,C} \leq 2/6\pi \\
& \vdots & \\
A, B \ {}_3\!\perp^1_{23} \ C & := & 0 < r_{A,B,C} \leq 1/3 \wedge 11/6\pi \leq \phi_{A,B,C} \leq 12/6\pi \\
A, B \ {}_3\!\perp^2_0 \ C & := & r_{A,B,C} = 1/3 \wedge \phi_{A,B,C} = 0 \\
& \vdots & \\
A, B \ {}_3\!\perp^3_0 \ C & := & 1/3 \leq r_{A,B,C} \leq 2/3 \wedge \phi_{A,B,C} = 0 \\
& \vdots & \\
A, B \ {}_3\!\perp^9_0 \ C & := & 3/2 \leq r_{A,B,C} \leq 3/1 \wedge \phi_{A,B,C} = 0 \\
& \vdots & \\
A, B \ {}_3\!\perp^{11}_{23} \ C & := & 3/1 \leq r_{A,B,C} \wedge 11/6\pi \leq \phi_{A,B,C} \leq 12/6\pi
\end{array}$$

This schema can be transferred and applied to arbitrary granularity $m$ of a calculus GPCC$_m$. The general segments $A, B \ {}_m\!\perp^i_j \ C$ are then so defined:

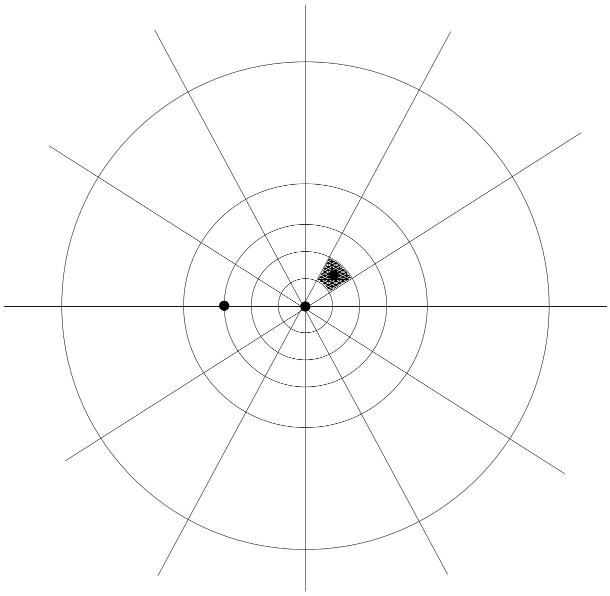

Figure 2: An example configuration of three points $A, B, C$. The depicted configuration corresponds to $A, B\ {}_3\!\perp^3_3$

$$
\begin{array}{rcl}
0 \leq j \leq 8m-2 \wedge j \mod 2 = 0 & \rightarrow & \phi_{A,B,C} = \frac{j}{4m}\pi \\
1 \leq j \leq 8m-1 \wedge j \mod 2 = 1 & \rightarrow & \frac{j-1}{4m}\pi < \phi_{A,B,C} < \frac{j+1}{4m}\pi \\
1 \leq i \leq 2m-1 \wedge i \mod 2 = 1 & \rightarrow & \frac{i-1}{2m} < r_{A,B,C} < \frac{i+1}{2m} \\
2 \leq i \leq 2m \wedge i \mod 2 = 0 & \rightarrow & r_{A,B,C} = \frac{i}{2m} \\
2m+1 \leq i \leq 4m-3 \wedge i \mod 2 = 1 & \rightarrow & \frac{m}{2m - \frac{i-1}{2}} < r_{A,B,C} < \frac{m}{2m - \frac{i+1}{2}} \\
2m+2 \leq i \leq 4m-2 \wedge i \mod 2 = 0 & \rightarrow & r_{A,B,C} = \frac{m}{2m - \frac{i}{2}} \\
i = 4m-1 & \rightarrow & m < r_{A,B,C}
\end{array}
$$

The idea of using qualitative ring-formed segments for the representation of knowledge of positions directly casts light on the integration of linguistic and navigational knowledge and is already in use in that field [5]. The problem is calculating the permutation and composition results for such structures by machine. The operation tables can be approximated with the aid of a composition of distance orientation intervals (DOI) [7]. Thereby flat segments and their borders are summarized. Thus one obtains thereby a quasi-partition in which only linear overlappings occur.

The tables for approximate transformations and for the approximate compositions of the calculi $\text{GPCC}_3$, $\text{GPCC}_4$, and $\text{GPCC}_5$ can be seen in the internet [12]. The calculi are, with respect to the transformation HMI, closed:

$$\text{HMI}\left( {}_m\bot_j^i \right) \quad = {}_m\bot_{8m-1-j}^{4m-i}$$

# 3 Application in Robotics Contexts

In robotic applications the relevant surface base relations with their borders are summarized into general relations. Out of this, one obtains a closed region in a plane (with the exception of its exterior segments which continue infinitely) as a relational proposition. The bounded line segments belong to both neighboring segments and points typically belong to four segments. All inner segments contain the point which corresponds to the relation $\mathrm{sam}$.

The surface area of these ambiguous acceptance areas is however 0. In the event that a corresponding border point triple is to be represented qualitatively, a disjunction of all bordering base relations must be used. As a result one obtains then a fine grained quasi-partition for the representation of the relative position of a point.

Already in the 1980s there were methods for representing uncertain position data in robotics using small acceptance areas with sharp boundaries [1]. However, this approach to modelling was not followed up on, as probabilistic modelling became available [11]. In laboratory situations in which systematic errors could be excluded through calibration methods, as a result only statistically independent measuring and movement errors remain, probabilistic approaches return very realistic estimates.

There the particularly favorable property of independently sourced errors will be used: they can mutually partially compensate each other. One then obtains, in contrast with propositions relying on fixed regions of error, more precise estimations. However, these estimations are optimistic and are not adequate when a pessimistic estimation is necessary for a critical application in the real world. As an example let us sketch the following scenario: A flying robot surveilles a bounded area. As soon as the danger arises that the robot leaves the area an emergency landing procedure would be required.

Here, an utterly pessimistic position estimate would be necessary in order to obtain the most reliable position estimation result. There, a probabilistic estimation can manage ordinary navigation. Only the recognition of an emergency and navigation will be managed by the pessimistic estimation.

## 3.1 Worst-Case-Estimation using a $1 - n\epsilon$ - Framework

To that end, an acceptance area will be defined such that the corresponding proposition with the probability $1 - \epsilon$ holds. A pessimistic global estimation (which does not exclude dependence) for $n$ propositions meets a probability of $1 - n\epsilon$.

# 4 Conclusion

We presented a calculus for representing and reasoning about qualitative relative orientation information. We identified systems of atomic relations on different granularity levels. The granularity of the calculus allows suppressing irrelevant feature changes in dynamically changing environments.

Potential applications of the calculus were motivated by a safe robotics scenario. In the scenario, pessimistic position estimate for safety critical applications is necessary.

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