Mechanics of Flow through Man-Made Lakes

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Man-made lakes are universally used to impound surface water for a great variety of purposes. In the Tennessee Valley Authority (TVA) region, there are about 30 major reservoirs (most of them created between 1930 and 1970) with a total water surface area of about 2400 km², a total water volume of about 27 billion m³, and depths of up to 137 meters. Water quality changes are known to occur in these water bodies and can be observed in the discharges for varying distances downstream. These changes are the result of complex interactions between many factors, such as geometry and size of the reservoir; storage management over the yearly cycle; outlet geometry and location; amount, distribution, and quality of the inflow; internal mixing processes; optical properties of the water; climate of the environment; biological and chemical processes in the water; and heat and mass transfer processes across the water surface and the ground [*Churchill and Nicholas*, 1967; *Wunderlich and Elder*, 1967].

The essential function of most reservoirs is the regulation of large temporary runoff for flood control and other purposes. The releases from a reservoir can be intermittent or relatively steady; sometimes these releases are rather large in comparison to natural river flows. The associated internal flow patterns can be very complex if density stratification exists. Heat transfer across the water surface and heat advection due to the inflows and outflows can cause a great variety of temperature and density patterns. Reservoirs with little inflow and outflow may maintain a considerable temperature difference between surface and bottom throughout the summer over a depth of only 10 meters, whereas strongly flushed reservoirs with deep intakes may be warm and homogeneous in temperature down to several times that depth [*Wunderlich and Elder*, 1968].

In the presence of a density gradient, water particles become stabilized at the elevation of their density, and sustained forces are needed to dislocate them permanently to other elevations since gravity or buoyancy forces tend to move them back to their original level. For example, the initial turbulence of inflow, no longer sustained by open-channel flow, dies down as the flow enters into the reservoir, and the water becomes quiescent. The reduction of turbulent vertical exchanges enables horizontal movements to persist over considerable distances. For the same reason, outflow is withdrawn from layers of limited thickness around the turbine intakes. Several distinct and independent currents may be simultaneously present within a reservoir, such as inflow currents, withdrawal currents, wind currents, and others [*Wunderlich and Elder*, 1967].

For several years, TVA has been conducting laboratory and field research to investigate the mechanics of flows in density-stratified reservoirs. The measurements of current velocities in reservoirs became possible with the development of a deepwater isotopic current analyzer that is capable of field use. This instrument, briefly called Dwica, has been described in detail elsewhere [*Vigander and Wunderlich*, 1971]. Its unique feature is a measuring range from 0.001 to 0.3 m/sec. The measurements are, however, very laborious and inherently slow, so the progress in data collection has been slow. It is not possible to obtain an instantaneous and complete picture of the velocity distribution in one cross section let alone in several cross sections at the same time. Therefore a considerable amount of speculation is still necessary in data interpretation. The use of dye injection in deep reservoirs has been explored and proved to be a means of identifying water masses, water movements, and, to some extent, dilution. The limited amount of data obtained

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so far by these techniques has provided, nevertheless, very valuable information on real prototype flow mechanics. In the following discussions, some of the Dwica and dye injection data are presented. For their interpretation, use is made of theoretical and laboratory test results obtained by others.

FLOW MECHANICS

In flow mechanics studies of stratified reservoirs the cause for density stratification must be known. Temperature differences are the most common cause, but other agents such as dissolved or suspended solids could be factors. In most TVA field tests, direct density measurements were made on water samples by using constant mass hydrometers [*Elder and Wunderlich,* 1968]. Apart from minor differences in absolute density the density gradient was always found to be directly related to temperature. A sample of these test results is shown in Figure 1. In the data analyses, pure water densities computed by the Thiessen-Scheel-Diesselhorst equation [*Tilton and Taylor*, 1937] at in situ temperatures are used.

Inflow. Dependent on the density difference between the incoming water and the reservoir water, overflow, underflow, or interflow may result. These flow patterns are shown schematically in Figure 2. Basically, any inflow seeks its density level and moves along this level into storage position. If this level happens to be the withdrawal zone, the inflow will move directly through the reservoir. In other cases, water may be stored for considerable periods of time.

Typical for all inflow types is the deepening of the inflow depth until a critical depth d_0 is reached. From this section downstream, underflow or overflow results depending on whether the density of the inflow is respectively greater than or less than that of the reservoir water. The depth of this critical section is governed by [*Harleman*, 1961, 1969]

$$F_0 = (Q/A_0)/[g(\Delta\rho/\rho)\ d_0]^{1/2} \qquad (1)$$

where Q is the inflow rate, A_0 is the cross-sectional area, g is the acceleration due to gravity, $\Delta\rho$ is the density difference between inflow and reservoir water, ρ is the density of the moving fluid, and d_0 is the critical depth. The critical section for an inflow of 19 m³/sec at 15°C into water of 28°C is shown in Figure 3. This section occurred between kilometer 4.5 and kilometer 4.4 on the Natahala River. The data yield a Froude number F_0 of about 0.5 at the critical section. Below this point the inflow moved under the warmer reservoir water to become an underflow, which can be followed in Figure 3 as it moved along the reservoir bottom.

If the inflow water is warmer than the reservoir

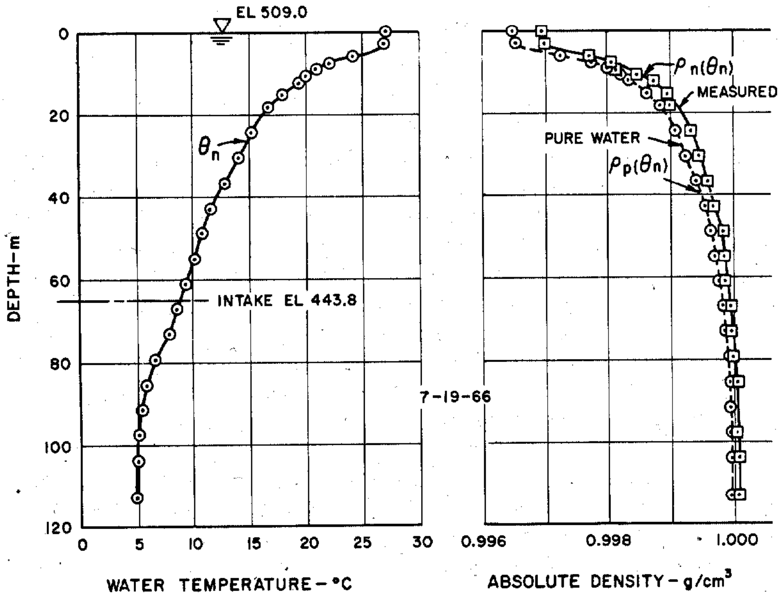
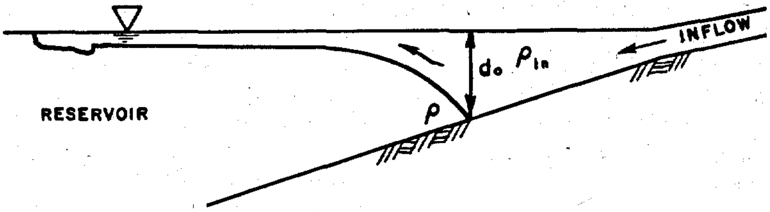
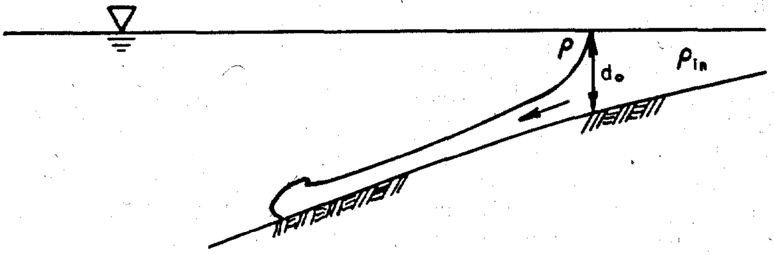


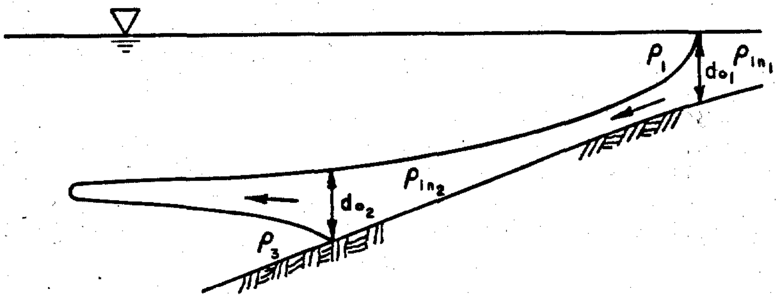
Fig. 1. Measured and pure water densities (Fontana).



(a) OVERFLOW $\rho_{in} < \rho$



(b) UNDERFLOW $\rho_{in} > \rho$



(c) INTERFLOW $\rho_{in_1} > \rho_1$, $\rho_{in_2} < \rho_3$

Fig. 2. Overflow, underflow, and interflow.

water, as is generally the case in spring, the inflow after passing the critical section spreads as a warm surface flow. An example of this type of flow is shown in Figure 4 where 16°C inflow water moved on top of 10°C reservoir water.

In deep reservoirs an underflow is likely to encounter still colder water, the result being that underflow ceases and interflow results. The level where this change occurs depends on the original inflow density and its modification due to mixing at the inflow point and along the underflow interface. At the interflow depth the flow depth increases and builds up the energy head necessary to drive the water along the plane of its density level and into its final storage position (Figure 2). An example of this type of flow is indicated by the dye concentration isopleths of Figure 5. In the upper part of the figure the inflow, originally at a mean temperature of about 14°C, arrives at the lift-off point in the vicinity of kilometer 125 on the Little Tennessee River at about 17°C. There the inflow deepens and feeds into its density layer. The downstream movement of the dye cloud shown in Figure 5 indicates a slow decrease in level elevation due to the withdrawal of water from the reservoir through an intake at an elevation of 444 meters.

Outflow. The location of the principal withdrawal intakes has a dominant influence on the water quality development in most reservoirs [*Wunderlich and Elder,* 1969]. Because of suppression of vertical movements in the presence of density stratification, water is withdrawn from a layer around the intake opening (Figure 6).

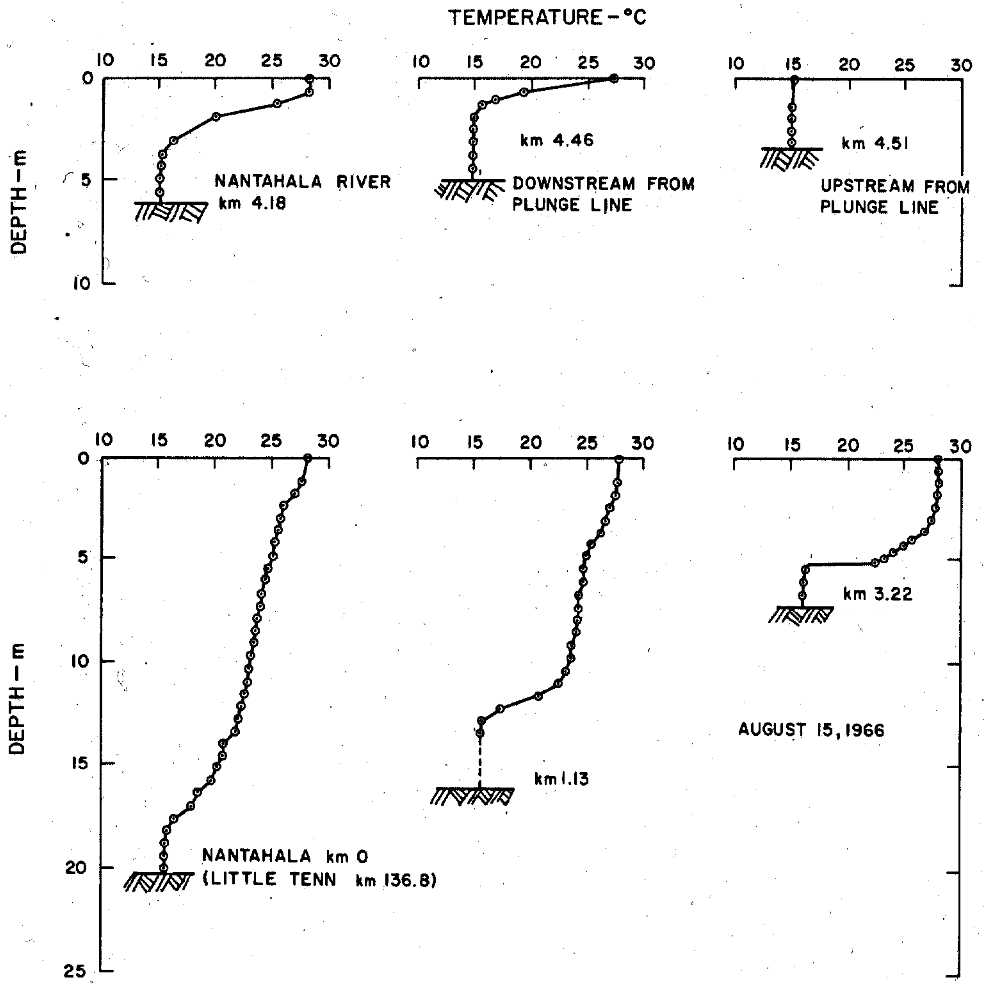


Fig. 3. Inflow. Plunging occurs near kilometer 4.5 (Fontana).

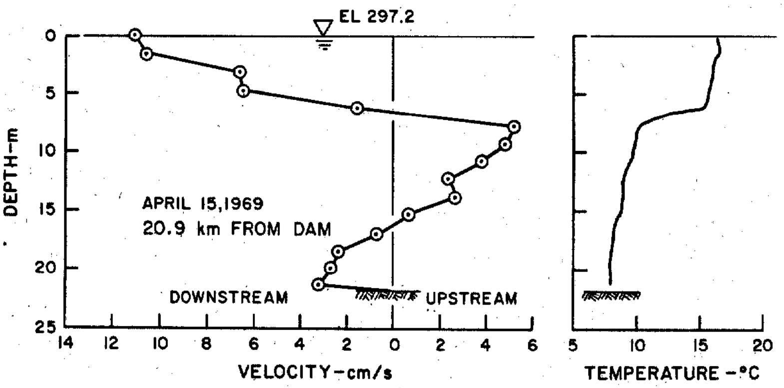


Fig. 4. Overflow of warm inflow (Douglas).

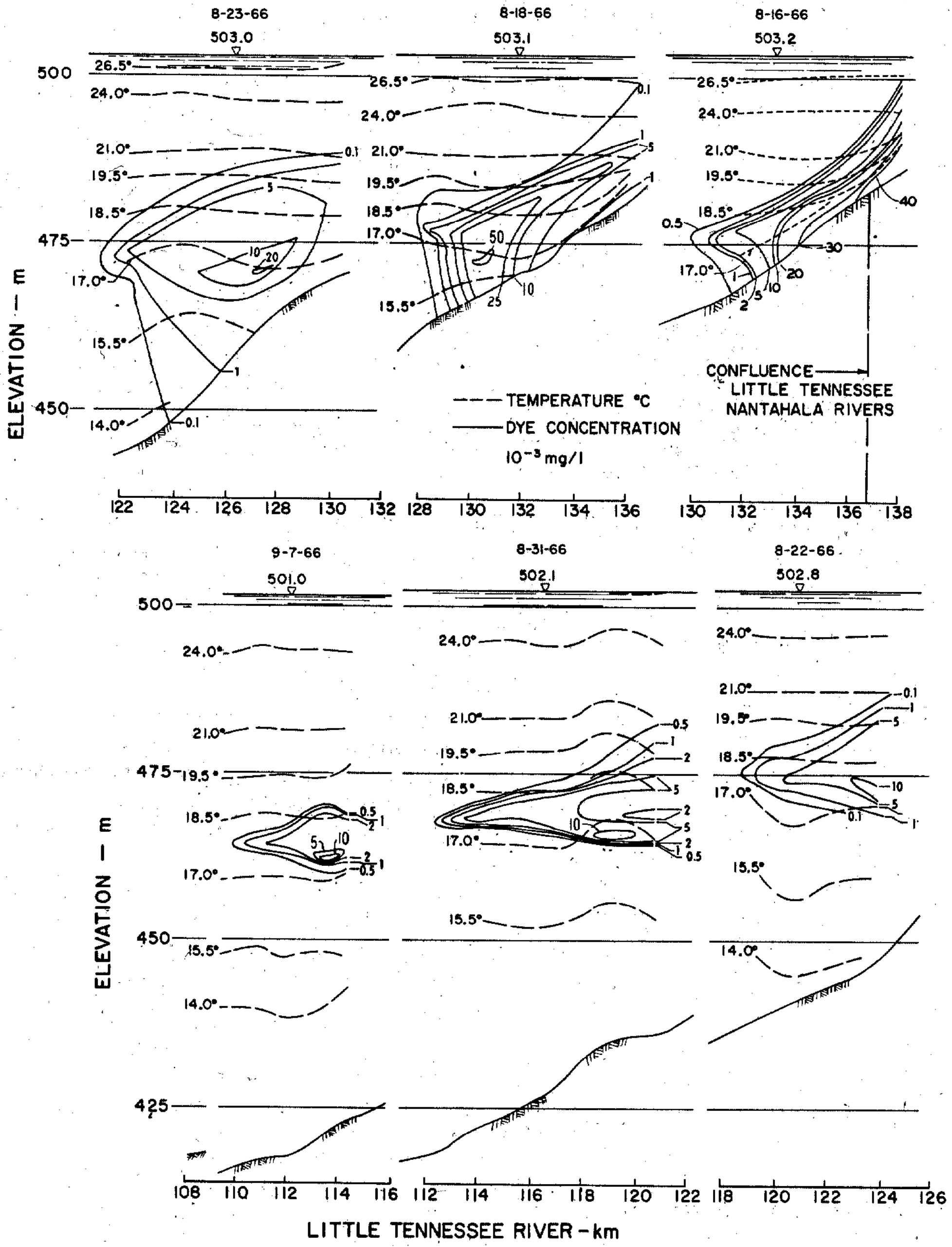
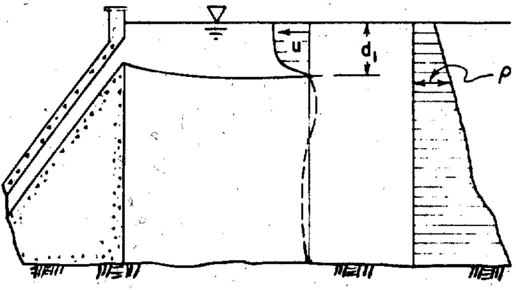


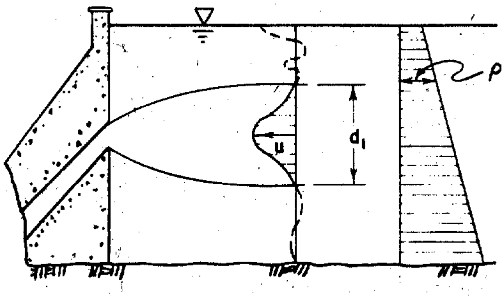
Fig. 5. Underflow and transition into interflow (Fontana).

Dependent on its location an intake may be classified as high, intermediate, or low. The determination of the thickness of the withdrawal layer for various intake configurations has been the subject of studies by several investigators and of field studies on several reservoirs.

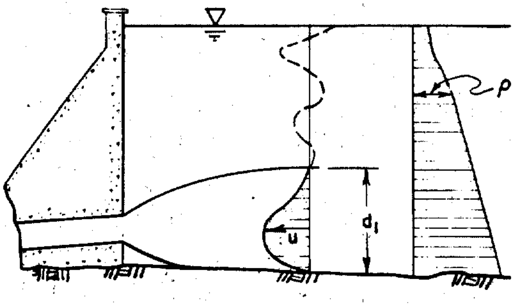
In Figure 7 the flow into a high intake is presented. In this case the cold outflow from an upstream impoundment at about 17°C moves on top of still colder water down the reservoir toward the intake. Solar heating creates a very thin warm layer of about 1 meter in thickness on



(a) HIGH INTAKE



(b) INTERMEDIATE INTAKE



(c) LOW INTAKE

Fig. 6. Selective withdrawal.

top of this flow, but this layer is also drawn into the intake.

In Figure 8 a rather warm summer inflow of 24°C moves toward the turbine intakes beneath a still warmer reservoir surface layer at 29°C. Below this current, cold winter water at 7°C is preserved as a stagnant pool throughout the summer.

A rather large flow into a low turbine intake is shown in Figure 9. Two velocity profiles measured at different times of the year but with similar density gradients and discharges of about 450 m^3/sec are presented. Other data pertaining to the test results in Figures 7, 8, and 9 are summarized in Table 1.

DISCUSSION OF RESULTS

Selective withdrawal from reservoirs can produce desirable or undesirable outflow water quality. For example, in deep reservoirs, cold water stored during the winter months can be very beneficial for municipal and industrial uses downstream. On the other hand, withdrawal of water from a layer with undesirable water quality (such as a low oxygen concentration) may cause problems downstream. Therefore it is important to have a criterion indicating whether selective withdrawal will occur and formulas for the prediction of the withdrawal layer characteristics.

Yih [1965] theoretically derived that no stagnant layers can exist when the densimetric Froude number of the total cross section is

$$F = (q/d^2)/(g\epsilon)^{1/2} \geq 0.33 \qquad (2)$$

where q is flow per unit width, d is total depth, g is acceleration due to gravity, $\epsilon = (1/\rho)\, d\rho/dz$ is the density gradient number that is positive for density increasing with depth, and ρ is the reference density and may be taken at the center line of the outlet opening. *Kao* [1970] concluded that for a given flow in a linearly stratified fluid the withdrawal layer thickness would grow to a size such that its Froude number becomes $F_1 = 0.33$; F_1 is similarly defined as F, and w_1, d_1, and $(d\rho/dz)_1$ refer to the withdrawal layer only. In a rectangular cross section with a constant density gradient, both Froude numbers are related by

$$F = (d_1/d)^2\, F_1 \qquad (3)$$

where d_1 is the withdrawal layer thickness, d is the total depth, and $F_1 = 0.33$, a universal constant, according to Kao. Hence the criterion for a stagnant layer to appear is $F < 0.33$. If $F > 0.33$, all layers move and contribute to the discharge. This theory was essentially confirmed by *Debler*'s [1959] model test results, which yielded $F_1 \approx 0.28$.

Withdrawal layer test results from 10 reservoirs are presented in Table 1. The Froude numbers are computed for the entire reservoir cross section at the indicated distances from the dam. They indicate that all layers still move at a Froude number of about $F = 0.06$, which is considerably less than 0.33. Two examples of rather uniform water movements in a stratified reservoir associated with $F = 0.066$ and $F = 0.159$ are shown in Figure 10. Because of cross-section changes, the measuring section used for the com-

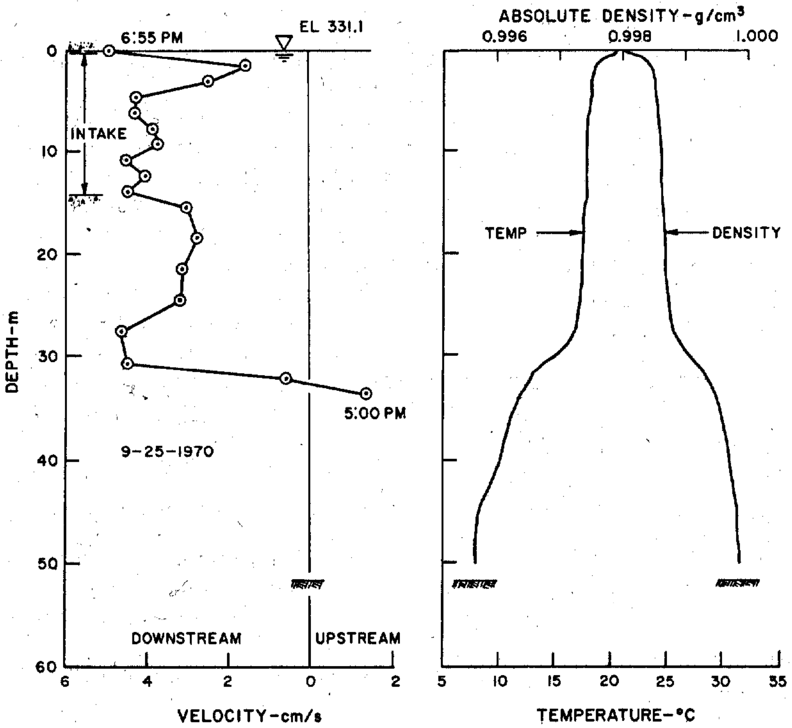


Fig. 7. Selective withdrawal through a high intake (Calderwood).

putation of F may not be the critical section that determines whether stagnant layers can exist or not. If, at the critical section, F reaches or exceeds the critical value, then the layers in all other sections must move to satisfy continuity, regardless of the Froude number. With all other parameters constant, such a critical section was assumed to exist close to the powerhouse and to run perpendicularly across an assumed streamline pattern. For both the Watts Bar and Fort Loudoun data

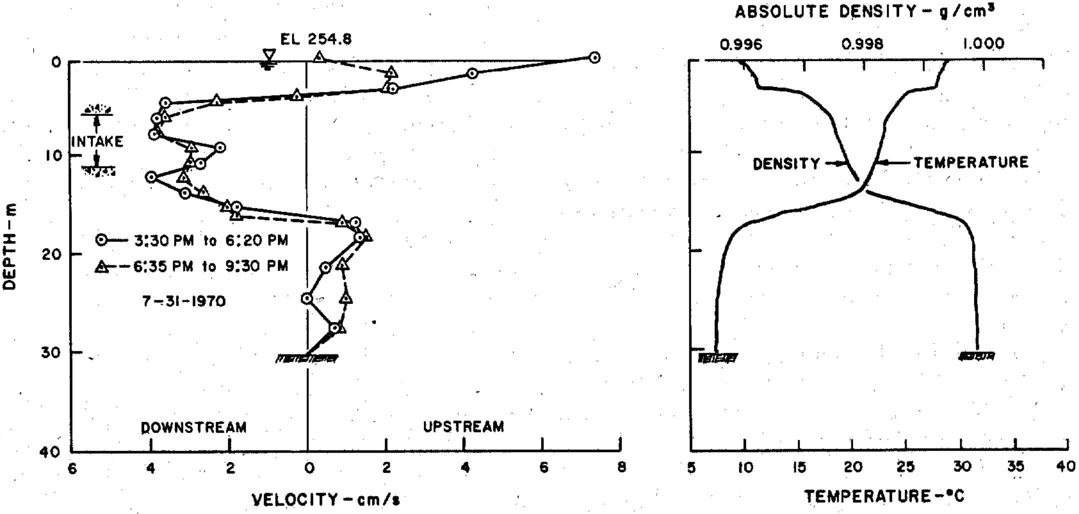


Fig. 8. Selective withdrawal through an intermediate intake (Ocoee 1).

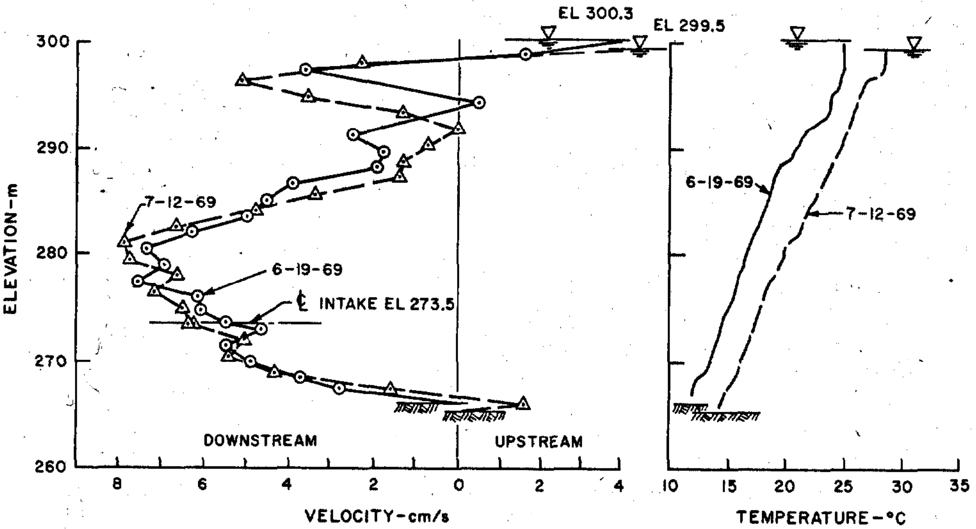


Fig. 9. Selective withdrawal through a low intake (Douglas).

the resulting reduction in width yielded Froude numbers of >0.33; this result would explain the movement of all layers in the measuring sections upstream.

For a two-dimensional linearly stratified flow the withdrawal layer thickness d_1 follows directly from (3) as

$$d_1 = (1/F_1)^{1/2} a \qquad (4)$$

wherein

$$a = F^{1/2} d = [q/(g\epsilon)^{1/2}]^{1/2} \qquad (5)$$

The term a was introduced by *Brooks and Koh* [1969] as 'scale length.' In natural reservoirs, flows are generally not two dimensional. Also, other flow in addition to withdrawal layer flow may be present in the cross section, and the density gradient may not be constant over the entire

TABLE 1. Froude Numbers of Reservoir Cross Section

Reservoir	Distance from Dam x, km	Depth d, meters	Area $10^{-3}A$, m^2	Gradient $10^4\epsilon$, m^{-1}	Flow Q_T, m^3/sec	Velocity Q_T/A, m/sec	$10^2(g\epsilon)^{1/2}$, sec^{-1}	Froude Number F*
Cherokee	2.7	44.8	46.0	0.732	311	0.007	2.68	0.006
Douglas	0.8	34.3	17.3	0.728	462	0.027	2.67	0.029
	0.8	33.8	16.4	0.916	430	0.026	3.00	0.026
	0.8	29.9	12.2	0.430	110	0.009	2.05	0.014
	0.8	34.1	17.1	0.988	227	0.013	3.12	0.012
	0.8	32.0	15.3	0.935	212	0.014	3.03	0.014
Fort Loudoun	3.2	22.9	9.3	0.420	799	0.086	2.03	0.185†
Melton Hill	1.2	20.7	4.2	1.063	275	0.065	3.23	0.097†
Watts Bar	0.6	24.4	11.7	0.591	453	0.039	2.41	0.066†
	0.6	22.6	11.5	0.846	1198	0.104	2.88	0.159†
Fontana	1.0	104.5	18.4	0.295	161	0.009	1.70	0.005
Calderwood	0.4	51.8	8.7	0.348	198	0.023	1.85	0.024
Chilhowee	1.6	16.5	5.0	1.148	207	0.041	3.36	0.075
Ocoee 1	0.8	30.2	10.7	1.302	68	0.006	3.58	0.006
Barren	1.0	23.2	13.9	0.971	78	0.005	3.09	0.008

*$F = [Q_T/(Ad)]/(g\epsilon)^{1/2}$, where Q_T is turbine discharge.
†All layers move.

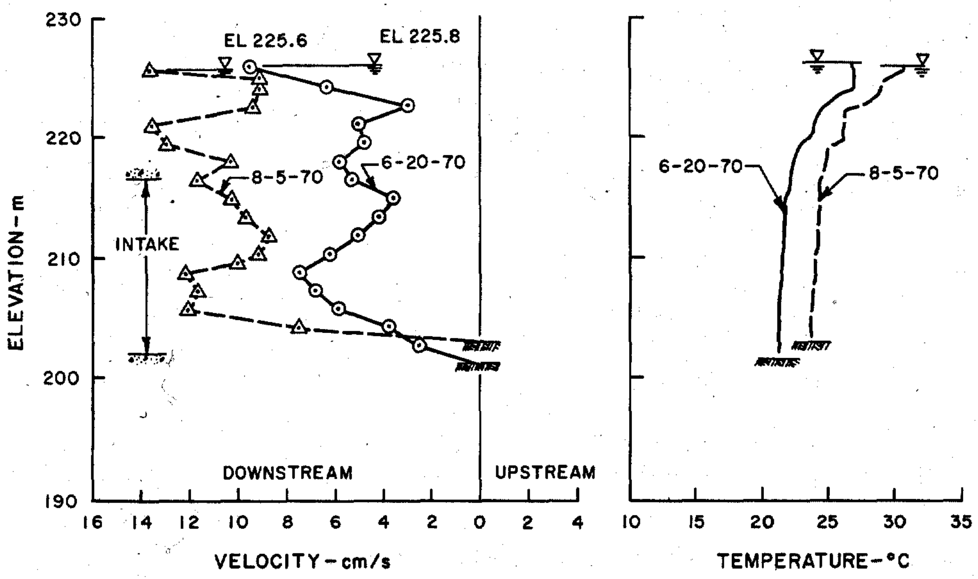


Fig. 10. Nonselective withdrawal through a low intake (Watts Bar).

depth. If it is assumed that under these different conditions the theoretical concepts can still be used, the withdrawal layer thickness can be expressed by

$$d_1 = (1/F_1)(Q_1/A_1)/(g\epsilon_1)^{1/2} \qquad (6)$$

Introducing $A_1 = w_1 d_1$ leads to

$$d_1 = (1/F_1)^{1/2} a_1 \qquad (7)$$

where $a_1 = [q_1/(g\epsilon_1)^{1/2}]^{1/2}$. The index 1 refers to the withdrawal layer. Equations 6 and 7 can be solved only by repeated trials, since all terms with index 1 are functions of d_1. For this reason, (6) was found to be convenient in practical applications. From the above discussion it also follows that d_1 should be evaluated in a narrow cross section and close to the intake (approximately within one reservoir width).

Values of $1/F_1$ and F_1 based on the field data and (7) are summarized in Table 2. A value of

TABLE 2. Withdrawal Layer Froude Number

Reservoir	Thickness d_1, meters	Area $10^{-3}A_1$, m^2	Gradient $10^4\epsilon_1$, m^{-1}	Flow Q_T, m^3/sec	Velocity Q_T/A_1, m/sec	$10^2(g\epsilon_1)^{1/2}$, sec^{-1}	Froude Number F_1*	$1/F_1$
Cherokee	29.6	20.4	0.696	311	0.015	2.61	0.020	50.4
Douglas	27.7	11.5	0.810	462	0.040	2.81	0.051	19.5
	25.1	10.1	0.879	430	0.043	2.94	0.057	17.4
	14.0	4.0	0.843	110	0.027	2.88	0.068	14.6†
	21.0	7.2	0.712	227	0.031	2.64	0.057	17.6
	19.4	6.6	0.945	212	0.032	3.03	0.055	18.2
Fort Loudoun	22.9	9.3	0.420	799	0.086	2.03	0.185	5.4†
Melton Hill	20.7	4.2	1.063	275	0.065	3.23	0.097	10.3
Watts Bar	24.4	11.7	0.591	453	0.039	2.41	0.066	15.2
	22.6	11.5	0.846	1198	0.104	2.88	0.160	6.3
Fontana	32.3	11.0	0.381	161	0.015	1.93	0.024	42.5
Calderwood	32.3	6.2	0.413	198	0.032	2.02	0.049	20.4
Chilhowee	13.1	3.8	0.466	207	0.055	2.14	0.194	5.1
Ocoèe 1	12.8	4.8	1.978	68	0.014	4.41	0.025	40.1†
Barren	14.9	7.4	1.486	78	0.011	3.82	0.018	54.2†

*$F_1 = [Q_T/(A_1 d_1)]/(g\epsilon_1)^{1/2}$, where Q_T is turbine discharge.
†Major discrepancy between turbine discharge and integrated withdrawal layer flow.

$1/F_1 \approx 41$, or $(1/F_1)^{1/2} = 6.4$, is obtained for intermediate intakes, such as Fontana. For surface and bottom intakes, $1/F_1 \approx 20$, or $(1/F_1)^{1/2} = 4.5$, seems to be appropriate, as is shown by the data for Douglas and Calderwood. Cherokee and Barren also have low intakes, but the factor $1/F_1$ turns out to be much larger than was expected, perhaps owing to the very irregular cross section in both cases and to a major departure from a linear density distribution in the second case [*Tennessee Valley Authority*, 1967, 1969*a*]. In contrast to these two latter cases, Douglas [*Tennessee Valley Authority*, 1969*b*] and Calderwood have rather regular cross sections.

As is indicated in Table 2, in some cases the flow rates obtained by integration of the velocity profiles did not satisfactorily match the turbine discharge to which the measured withdrawal layer velocities were attributed. The discrepancies between integrated flow and measured turbine flow are in part due to the fact that the flow through the measuring cross section is related to the water balance of the reservoir between the measuring section and the dam rather than to turbine discharge alone. Therefore Table 2 was recomputed, and integrated flow rates instead of measured turbine discharges were used. The resulting Froude numbers for the principal moving layer did not substantially change the results of Table 2.

Based on inviscid theory of stratified flow into a bottom slot [*Brooks and Koh*, 1969] the withdrawal layer thickness can be expressed by (7). Adaptation of this solution to an intermediate intake [*Brooks and Koh*, 1969] yields

$$d_1 = (1/F_1)^{1/2}(2)^{1/2}a_1 \qquad (8)$$

Hence, if (7) and (8) are used, $(1/F_1)^{1/2} = 4.5$ is the same for all intakes.

At the present time the true causes for the low values of F_1 for prototype withdrawal layers are not known. The above discussion may point at some of them. It has also been suggested that the presence of turbulent heat diffusivity may be one cause [*Brooks and Koh*, 1969; *King*, 1969]. In addition, the internal velocity and flow distribution in the withdrawal layer [*Elder and Wunderlich*, 1969; *Wunderlich and Elder*, 1970], intake geometry, nonlinearity of density gradients [*Bohan and Grace*, 1970], and the presence of other flows besides the withdrawal flow may be factors.

CONCLUSIONS

Theoretical results as well as laboratory and field data are used to interpret the mechanics of density-stratified reservoirs. Many problems remain to be clarified, especially those related to water movements of inflows, including all categories such as overflow, underflow, and interflow. Discrepancies between theoretical and field results concerning selective withdrawal also require further investigations to pinpoint their true causes. If predictions are required, (7) is suggested for bottom and surface withdrawal intakes and (8) is suggested for intermediate intakes by using $(1/F_1)^{1/2} = 4.5$ in both cases. These equations and the experimentally modified Froude number $F_1 \approx 0.05$ should account to some extent for geometries and flow characteristics generally encountered in real reservoirs.

NOTATION

A,	total cross-sectional area of the reservoir, m^2;
A_1,	cross-sectional area of the withdrawal layer, m^2;
A_0,	cross-sectional area of the reservoir at critical section where inflow becomes overflow or under flow, m^2;
a,	scale length after *Brooks and Koh* [1969] for the entire cross section, meters;
a_1,	scale length with q_1 and ϵ_1 for the withdrawal layer only, meters;
d,	maximal depth of total cross section, meters;
d_1,	thickness of withdrawal layer, meters;
d_0,	depth of the critical cross section A_0, meters;
g,	acceleration due to gravity, m/sec^2;
F,	Froude number of total reservoir cross section, equal to $(Q/Ad)/(g\epsilon)^{1/2}$;
F_1,	Froude number of withdrawal layer only, equal to $[Q_1/(A_1d_1)]/(g\epsilon)^{1/2}$; a universal constant, equal to 0.33 after *Kao* [1970];
F_0,	Froude number of the critical inflow section where overflow or underflow starts, assumed to be about 0.5;
Q,	total flow in reservoir cross section, m^3/sec;
Q_1,	withdrawal layer flow, m^3/sec;
Q_T,	total turbine discharge, m^3/sec;
q,	flow per unit width in total cross section, equal to Q/w, m^2/sec;
w,	mean width of total reservoir cross section, defined as $w = A/d$, meters;
w_1,	mean width of the withdrawal layer, defined as $w_1 = A_1/d_1$, meters;
ϵ,	density gradient number, in stably stratified fluids always positive, equal to $(1/\rho)\, d\rho/dz$, 1/m;
ϵ_1,	density gradient number for withdrawal layer only, equal to $(1/\rho)\,(d\rho/dz)_1$, 1/m;
$\Delta\rho$,	density difference, in stably stratified fluids always positive, kg/m^3.
ρ,	reference density, a representative density of the moving fluid, inflow density in (1), the density at intake center line in the gradient numbers ϵ and ϵ_1, kg/m^3.

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