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SEASONAL AND LONG-TERM VARIATIONS OF DISSOLVED SOLIDS IN LAKES AND RESERVOIRS

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ABSTRACT: The seasonal and long-term variations of dissolved solids in lakes and reservoirs are analyzed in accordance with the hydrologic- and mass-balance equations. The volume and surface area are expressed as power functions of depth that yield analytical solutions for the long-term analysis. The equation defining the water elevation and volume may be approximated by exponential functions that simplify the solution of the mass-balance equation of the dissolved solids. For the intermediate and seasonal time scales, periodic and exponential functions define the hydrologic components, providing the forcing functions for the dissolved solids equations. For both cases, an exponential residence time transforms the mass-balance equation, leading to analytical solutions. Given the mass and volume, the concentration follows. The temporal variations of dissolved solids, calculated by the associated mass and volumetric equations, are compared to the observed change in salinity in lakes and reservoirs of various geophysical and hydrological characteristics.

INTRODUCTION

Increasing concentration of dissolved solids due to irrigation returns, industrial uses, and road-deicing practices has been observed in many water bodies throughout the country. This condition, which is further aggravated in areas where evaporation exceeds precipitation and runoff, has long been recognized (Bubeck 1971; Hely et al. 1966) and has recently been studied with respect to diversion for water supply ("Mono Basin Ecosystem" 1987; "Mono Basin Geology" 1987). Furthermore, the increase in evaporation, expected to accompany greater atmospheric temperatures ("Changing" 1983), may cause a decrease in freshwater volume and thus exacerbate this and other water-quality problems.

It is the purpose of this paper to present an analysis of the long-term and seasonal variation of dissolved solids in lakes and reservoirs. The basic equations are developed from the continuity principle, which applies to both the volume of water and the mass of dissolved solids. The general approach consists of first developing the hydrologic balance of the water body, from which the temporal variation of the volume is established. A mass balance of the dissolved solids is next constructed, incorporating the relevant elements of the water balance. The concentrations that follow from the mass and hydrologic equations are compared to the observed changes in salinity in lakes and reservoirs of various geophysical and hydrologic characteristics.

HYDROLOGIC BALANCE

Consider a lake or reservoir of volume V receiving freshwater inflow I , which includes the discharge of rivers entering the lake and direct runoff from the tributary drainage area. Also take into account precipitation P , evaporation E , groundwater seepage G , and outflow Q . The change in vol-

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ume is the net effect of the temporal variations of these components:

$$\frac{dV}{dt} = I - Q + P - E \pm G \dots \dots \dots (1)$$

The last term of Eq. 1 may be positive or negative in accordance with the discharge or recharge of the groundwater reservoir. Municipal, industrial, or agricultural effluents may be additional factors to take into account. The precipitation and evaporation in Eq. 1, divided by the surface area A yields $\rho = PA^{-1}$ and $\epsilon = EA^{-1}$, respectively, in which the units of ρ and ϵ are depth per unit time, the customary manner of reporting these data.

An equilibrium is achieved when the positive and negative components of Eq. 1 are equal. Although this condition is rarely encountered over a seasonal time scale, due to the rainfall-runoff variation and the characteristic temperature periodicity over the year, it may be approximated over longer time scales. Both equilibrium and time-variable conditions are addressed.

Eq. 1 is most expeditiously solved by expressing the volume and surface area in terms of the depth:

$$V = c_1 y^m \dots \dots \dots (2)$$

$$A = c_2 y^n \dots \dots \dots (3)$$

Eqs. 2 and 3, in which y = the maximum depth, may also be expressed in terms of the average depth $H = V/A$. Analysis of depth and volume measurements indicate values of $1 < m < 3$ (Hutchinson 1957). Substituting Eq. 3 for the area and the time derivative of Eq. 2 for the volume, Eq. 1 becomes

$$c_1 m \frac{dy}{dt} = \frac{\Delta_1}{y^n} + c_2 \Delta_2 \dots \dots \dots (4)$$

in which $n = m - 1$; $\Delta_1 = I - Q \pm G$; and $\Delta_2 = \rho - \epsilon$. For the initial condition $y = y_0$ ($H = H_0$, $A = A_0$, and $V = V_0$), the solution of Eq. 4, with Δ_1 and Δ_2 constant, is for $m = 2$ and $n = 1$:

$$\frac{y}{y_0} - \frac{\Delta_1}{\Delta_2} \left[\ln \left(1 + \frac{\Delta_2 y}{\Delta_1 y_0} \right) - \ln \left(1 + \frac{\Delta_2}{\Delta_1} \right) \right] = 1 + \frac{\Delta_2 t}{2H_0} \dots \dots \dots (5a)$$

in which

$$\frac{y}{y_0} = \frac{A}{A_0} = \left(\frac{V}{V_0} \right)^{1/2} \dots \dots \dots (5b)$$

and for $m = 3$ and $n = 2$:

$$\frac{y}{y_0} - \left(\frac{\Delta_1}{\Delta_2} \right)^{1/2} \left[\tan^{-1} \left(\frac{\Delta_2}{\Delta_1} \right)^{1/2} \frac{y}{y_0} - \tan^{-1} \left(\frac{\Delta_2}{\Delta_1} \right)^{1/2} \right] = 1 + \frac{\Delta_2 t}{3H_0} \dots \dots \dots (6a)$$

in which

$$\frac{y}{y_0} = \left(\frac{A}{A_0} \right)^{1/2} = \left(\frac{V}{V_0} \right)^{1/3} \dots \dots \dots (6b)$$

Eqs. 5 and 6 are appropriate for long-term analyses (decades), in which the components of the hydrologic balance, Δ_1 and Δ_2 , may be represented by long-term averages. Numerical evaluation of Eqs. 5 and 6 for various combinations of Δ_2/Δ_1 and Δ_2/mH_0 , indicates that a practical approximation is

$$y - y_0 e^{-\Delta_2 t} \dots \dots \dots (7)$$

Substitution of Eq. 7 in Eq. 2 yields

$$V = V_0 e^{\lambda t} \dots \dots \dots (8)$$

in which $V_0 = c_1 y_0^m$, and $\lambda = mh$, with h defined by Eq. 7.

For the seasonal time scale (months), and possibly the intermediate scale (years), the temporal variation of the inflow, evaporation, and rainfall are significant. Eqs. 5 and 6 are evidently not appropriate, and Eq. 2 is solved directly

$$V = \int_0^t (\Sigma J - \Sigma O) dt \dots \dots \dots (9)$$

in which ΣJ and ΣO = summations of the inflows and outflows, respectively.

MASS BALANCE

Each of the hydrologic components, with the exception of the evaporation, may be a source of dissolved solids, a mass balance of which yields

$$\frac{dM}{dt} = \frac{d(Vc)}{dt} = \Sigma W - Qc \dots \dots \dots (10)$$

in which $\Sigma W = I c_i + P c_p + G c_g + W_e$. The subscript of the concentration identifies the associated hydrologic component, and W_e refers to the effluent mass rates from municipal, industrial, and agricultural sources. The concentration is a spatial and temporal average over the seasonal or annual period, during which the effect of thermal stratification on the concentration is minimal compared with the other factors in the mass balance. The equations, developed on this basis, are appropriate for dissolved conservative constituents, which are less affected by stratification than nonconservative or particulate substances. For reservoirs, which are highly stratified for extended periods, the outflow concentration must be modified to account for the difference in concentration between the epilimnion and hypolimnion. For moderately stratified reservoirs and closed lakes, however, the equations are reasonable approximations, as discussed later.

Introduction of the volume in the last term of Eq. 10 transforms the concentration to mass, which yields

$$\frac{dM}{dt} + \phi_1(t)M = F_1(t) \dots \dots \dots (11)$$

in which $\phi_1(t) = Q/V$; and $F_1(t) = \Sigma W$; $\phi_1(t)$ = the reciprocal of the detention or residence time t_d . The solution of Eq. 11 defines the temporal variation of mass in the water body $M(t)$, which, divided by $V(t)$, evaluated by the hydrologic balance, yields the concentration $c(t)$.

An alternate form, in terms of the concentration, evolves by expanding the left-hand side of Eq. 10 and dividing through by the volume (O'Connor 1976):

$$\frac{dc}{dt} + [\phi_1(t) + \phi_2(t)]c = F_2(t) \dots \dots \dots (12)$$

in which $\phi_2(t) = V^{-1} dV/dt$; and $F_2(t) = \Sigma W/V$.

Eqs. 11 and 12 are the general forms of the mass-balance equations. In many cases, they may be expressed as first-order differential equations, and solution by numerical methods is straightforward. Under certain mathematical conditions that have realistic physical counterparts, these equations may

be integrated directly. Analytical solutions usually require less computational effort and generally provide greater insight and understanding of the phenomena.

For a specific geophysical setting and problem context, the general equations take on a particular form that reflects the morphology of the basin, the hydrological elements of the drainage area, and relevant sources and sinks of dissolved solids. With respect to the geomorphological characteristics, lakes may be classified as closed (seepage) or open (drainage) (Hutchinson 1957). The former has no surface-outlet channel, and the water loss occurs primarily by evaporation, characteristic of many lakes in the western part of the United States. The latter is defined by a topography that permits water to flow from the lake through an effluent channel, typical of most lakes in the eastern section of the country.

This classification also provides the basis for defining the limiting values of the coefficients ϕ_1 and ϕ_2 . In closed lakes, the residence time approaches infinity, and its reciprocal ϕ_1 approaches zero. This condition reduces the mass balance (Eq. 11) to a simple differential equation. Furthermore, the temporal variation of the volume may be approximated by an exponential function (Eq. 8). It follows, then, that ϕ_2 is constant, since the derivative of Eq. 8 is

$$\frac{1}{V} \frac{dV}{dt} = \pm v = \phi_2 \dots \dots \dots (13)$$

For closed lakes, therefore, $\phi_1 = 0$ and $\phi_2 = \pm v$.

The residence time of open lakes is a temporal variable due to the change in volume and outflow. Compared with closed lakes, the volume exhibits less pronounced variations, due to the modulating effect of the outflow. Since the volume and outflow of open lakes and reservoirs are functionally related, the variation of the residence time from year to year is minimal, and as a limiting condition approaches a constant value, defined by the long-term mean. The coefficients for open lakes may thus be approximated by $\phi_1 = t_d^{-1}$ and $\phi_2 = 0$.

For each of these limiting conditions, the solution of the concentration form of the mass balance (Eq. 12) for the long-term analysis is

$$c(t)e^{\pm \alpha t} = c_0 + \int F_2(t)e^{-\alpha t} dt \dots \dots \dots (14)$$

The exponential coefficient $\alpha =$ the volumetric coefficient $\pm v$ for closed lakes, or the reciprocal of the residence time t_d^{-1} for open lakes and reservoirs. Eq. 14 lends itself to direct solution for a variety of forcing functions $F_2(t)$ that define the temporal variation of the mass inputs (Chapra and Reckhow 1983; O'Connor and Mueller 1970). For constant mass input W , the solution of Eq. 14 for closed lakes is

$$c = \frac{W}{V_0} t e^{\pm \alpha t} + c_0 e^{\pm \alpha t} \dots \dots \dots (15)$$

in which the positive and negative exponents refer to decreasing and increasing volumes, respectively. For open lakes, the solution of Eq. 14 is

$$c = \frac{W t_d}{V} (1 - e^{-t/t_d}) + c_0 e^{-t/t_d} \dots \dots \dots (16)$$

The significance of the outflow in the hydrologic- and mass-balance equations is to be noted. If its magnitude is small compared with the remaining four factors in the hydrologic balance (Eq. 1), its precise enumeration may

not be necessary to define the volume. On the other hand, the outflow invariably takes on more importance in the mass balance (Eq. 11 or Eq. 12). The outflow, with the volume, defines the residence or detention time, the reciprocal of which is associated with the dependent variable. The specification of the volume, outflow, and time scale, which is used to evaluate the detention with respect to the time scale of the water-quality problem, are key steps in the analysis.

TIME VARIABLE FLOW AND VOLUME

For the more general case, the outflow Q and the volume V are temporal variables, with a ratio of $\phi_1(t)$. While various mathematical expressions of this parameter lead to analytical solutions of Eq. 11, a particularly amenable form is the exponential function. Since the volume may be approximated (Eq. 8), it is reasonable to assume a similar approximation for the residence time:

$$t_d = t_{d0} e^{-at} \dots \dots \dots (17)$$

in which $t_{d0} = V_0/Q_0$ where 0 = the initial condition of each variable. After substitution of Eq. 17, Eq. 11 becomes

$$\frac{dM}{dt} + \frac{e^{-at}}{t_{d0}} M = \Sigma W \dots \dots \dots (18)$$

Transformation of the time coordinate to $\tau = e^{-at}$ and substitution of the transform and its derivative yields

$$\frac{dM}{d\tau} \pm \frac{M}{at_{d0}} = \frac{\Sigma W}{a\tau} \dots \dots \dots (19)$$

The positive and negative signs define decreasing and increasing residence times, respectively. For the former case, the general solution is

$$M e^{-\tau/at_{d0}} = \frac{1}{at_{d0}} \int \frac{e^{\tau/at_{d0}}}{\tau} W(\tau) d\tau + C \dots \dots \dots (20)$$

The explicit solution is obtained by completing the quadrature and evaluating the constant, C by the initial conditions. For constant W , the solution is

$$M e^{-\beta} = \frac{W}{a} \left[\ln at_{d0}\beta + \beta \left(1 + \frac{\beta}{2 \cdot 2!} + \frac{\beta^2}{3 \cdot 3!} + \dots \right) \right] + C \dots \dots \dots (21)$$

in which $\beta = \tau/at_{d0}$. The initial condition ($M = M_0$ at $\tau = 0$, $t = 1$) for $\beta < 0.1$, leads to

$$M = \frac{W}{a} e^{-\tau/at_{d0}} \left[at + \frac{1}{at_{d0}} (e^{at} - 1) \right] + M_0 e^{-(1/at_{d0})(e^{at}-1)} \dots \dots \dots (22)$$

The general solution for the increasing function of detention time is given by Eq. 20, with the signs of the exponential powers reversed. A closed-form solution for this case is not readily available. However, a close approximation for both functions may be effected by virtue of the characteristics of the integrand in Eq. 20. Over a period Δt for which the average detention time is t_d

$$\int \frac{e^{\pm\tau/at_{d0}}}{\tau} d\tau \approx \int e^{\pm(1/at_{d0}) + (1/t_d)} dt \dots \dots \dots (23)$$

The positive and negative signs of the exponent refer to the decreasing and increasing functions, respectively. Substitution of Eq. 23 in Eq. 20 and integration for constant W yields

$$M = W\bar{i}_d e^{-\phi}(e^{t/\bar{i}_d} - 1) + M_0 e^{-\phi} \dots (24)$$

For decreasing detention

$$\phi = \frac{e^{at} - 1}{at_{d0}} \dots (25a)$$

$$\bar{i}_d = t_{d0} \frac{1 - e^{-a\Delta t}}{a\Delta t} \dots (25b)$$

For increasing detention

$$\phi = \frac{1 - e^{-at}}{at_{d0}} \dots (26a)$$

$$\bar{i}_d = t_{d0} \frac{e^{a\Delta t} - 1}{a\Delta t} \dots (26b)$$

The average detention time \bar{i}_d is evaluated over the interval Δt . For $at \ll 1$, ϕ reduces to $\sim t/t_d$ with t_d approaching t_{d0} , and Eq. 24 becomes

$$M = W\bar{i}_d(1 - e^{-t/\bar{i}_d}) - M_0 e^{-t/\bar{i}_d} \dots (27)$$

Eq. 27 is equivalent to Eq. 16, with \bar{i}_d defined by Eqs. 25b and 26b for decreasing and increasing detention functions, respectively.

APPLICATIONS

The preceding equations are applied to both steady- and nonsteady-state hydrologic conditions, in conjunction with various sources of dissolved solids. The examples relate to the long-term and seasonal effects of deicing salts on the chloride concentrations in Irondequoit Bay, New York. The change in salinity due to surface inflow and irrigation drainage in the closed basin of Salton Sea, California; the effect of impoundment on water quality in Cheney Reservoir, Kansas; and the increase in salinity in Mono Lake, a closed basin in California.

Irondequoit Bay

The use of salt for the deicing of roads and highways was initiated in the 1930s, and since the late 1940s has increased sharply. This practice has had dramatic effects on the quality of waterways that drain these areas, a notable example of which is Irondequoit Bay near Rochester, New York (Bubeck et al. 1971). In the early part of the century, the increase in chloride concentration in the bay was due to municipal and possibly some industrial or commercial usage, which presumably follows the population growth, as presented in Fig. 1(a). Deicing began in 1940, when this source was approximately equal to the municipal and industrial use, increasing by a factor of twenty in 1970, as shown in Fig. 1(b). Exponential functions are used to define the mass rates of inputs of both the municipal and deicing sources.

$$W = W_0 e^{mt} \dots (28)$$

The municipal mass rate is based on a population of 60,000 in 1900, with a water consumption of 100 gal (0.379 m³) per capita day and a chloride concentration of 75 mg/L ($W_0 = 622$ T/yr).

Data on salt use for deicing in the drainage basin for the period from 1966 to 1970 indicate an average basin-town ratio of three. This ratio is applied to the preceding years, 1955–1965, when data for the town was available. The relationship presented in Fig. 1(b) for the deicing use is based on $W_0 = 240$ T/yr. Furthermore, Bubeck et al. (1971) estimated that about 50%

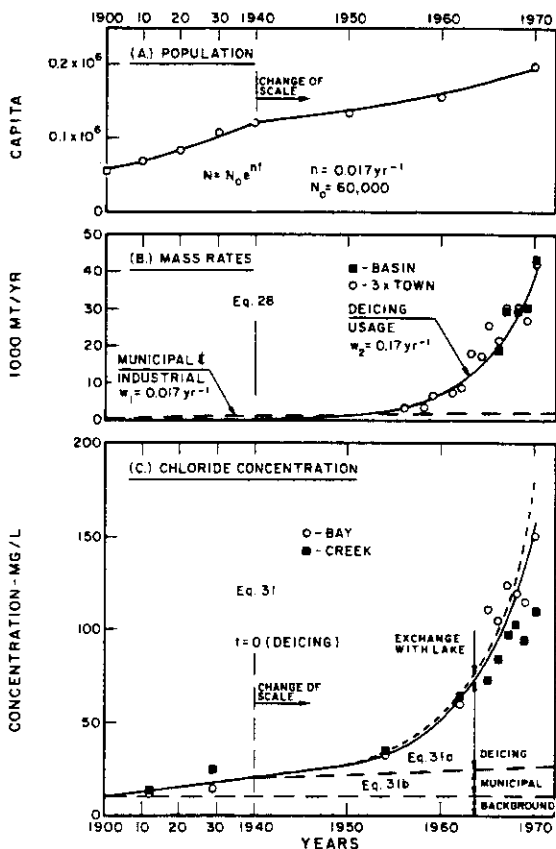


FIG. 1. Irondequoit Bay: Chloride Concentration—Long-Term

of the salt used for deicing is stored in the soil and groundwater, and the remaining 50% is discharged to the bay during the year. Thus, the value of W_0 in the mass-balance equation for the bay is 120 T/yr.

The equation for the chloride concentration from each of the sources for a constant detention time, in general accordance with Eq. 12, is

$$\frac{dc}{dt} + \frac{c}{t_d} = \frac{W_0 e^{wt}}{V} - R(c - c_L) \quad \dots \dots \dots (29)$$

in which R = fractional exchange of bay volume per unit time; and c_L = concentration in Lake Ontario. The second term on the right-hand side of Eq. 29 takes into account the exchange due to the concentration gradient between the bay and the lake.

The solution of Eq. 29 is

$$c = \frac{W_0}{\left(\frac{1}{t_d} + w + R\right)V} (e^{wt} - e^{-\kappa(1/t_d+R)t}) + \frac{Rc_L}{\frac{1}{t_d} + R} (1 - e^{-\kappa(1/t_d+R)t}) + c_0 e^{-\kappa(1/t_d+R)t} \quad \dots \dots \dots (30)$$

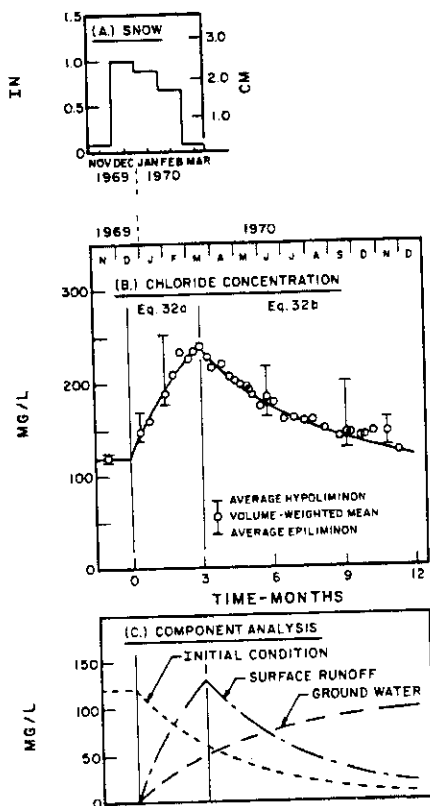


FIG. 2. Irondequoit Bay: Chloride Concentration—Seasonal

The exchange R is estimated by $\Delta H/H$, in which ΔH is the water-level differential induced by tidal, hydrological, and meteorological effects. The average depth H is about 7 m. The order of the tidal amplitude in the Great Lakes is 1 cm per day, and the seasonal variation of water level in the bay is about 0.5 m (Hutchinson 1957). A range of the exchange coefficient may thus be estimated as 0.1–0.5/yr. The resulting mass transfer, the second term of Eq. 30, is therefore insignificant compared with mass inflows from the municipal and deicing sources. The residence time, based on the mean annual flow (4.0 m³/s) and volume (0.046 km³), is 4.4 months ($t_d^{-1} = 2.7 \text{ yr}^{-1}$). The exponential term, containing the residence time, therefore becomes insignificant within a few years. For the time scale of the analysis (70 years), Eq. 30 reduces to

$$c = \frac{W_0 e^{wR}}{\left(\frac{1}{t_d} + w + R\right)V} \dots \dots \dots (31)$$

The concentrations calculated by Eq. 31 for the municipal source—with $t = 0$ in 1900, the deicing $t = 0$ in 1940, and the total concentration of chlo-

ride, which includes a background level (10 mg/L), assigned as that observed in 1900—are shown in Fig. 1(c). The effect of exchange with Lake Ontario ($R = 0.5 \text{ yr}^{-1}$) is shown by the dashed line. The data are surface summer observations.

The deicing salt that is usually distributed in December, January, and February, in accordance with the snow precipitation, is shown in Fig. 2(a). In 1969–70, 46,000 T of chloride were used for deicing within the Irondequoit drainage basin. Following the estimate noted earlier, 50% of this amount entered the bay through surface runoff and groundwater inflow. Assuming that a surface runoff of 9,000 T is distributed over three months and the groundwater inflow of 14,000 T over the year, the average monthly inflows are 3,000 (W_s) and 1,170 (W_g) T, respectively. With these values as constants over the periods indicated, the solution is of the form of Eq. 16:

$$c_1 = \frac{W_s + W_g}{Q} (1 - e^{-t/s}) + c_{01} e^{-t/s} \dots \dots \dots (32a)$$

$$c_2 = \frac{W_g}{Q} (1 - e^{-t/g}) + c_{02} e^{-t/g} \dots \dots \dots (32b)$$

in which s and g refer to the surface and groundwater components of the mass inflow; the initial concentration $c_{01} = 120 \text{ mg/L}$, the vertically uniform concentration after the fall turnover in November; and the concentration $c_{02} = c_1$ at $t = 3$ months. The chloride concentration in accordance with Eq. 32 and the component contributions are shown in Figs. 2(b) and 2(c), respectively. The data are volume-weighted means with representative average concentrations in the epilimnion and hypolimnion. The latter is a small fraction of the total volume and, thus, the concentration in the upper layer approaches the weighted average.

It is evident from the preceding discussions that the critical assumption is the distribution between the surface and ground mass inflows, as well as the periods over which they are effective. The values assigned are not unique and other combinations of the mass and time parameters may fit the observations as well. Furthermore, the variability of the surface and groundwater inflows are not taken into account. The following applications address this issue.

Salton Sea

Salton Sea is located in the southeastern corner of California. The sea was formed in the early part of the century as a result of the diversion of the Colorado River for irrigation and the occurrence of unexpected floods. In 1904, almost the entire flow of the Colorado River was diverted into the Salton Sea basin. This diversion was out of control for more than two years until February 1907. During the following decades, the water level receded until the trend reversed in about 1924.

Although Salton Sea is suitable for neither human consumption nor irrigation because of its high salinity, it has been an important recreational resource for the metropolitan regions of southern California. In 1961–1962, a study of the sea was conducted (Hely et al. 1966). The inflow, precipitation, evaporation, and concentrations of salinity were measured, and evaporation was computed by water and energy budgets and estimated by mass-transfer equations. The relevant hydrological data are presented in Fig. 3(a) (Hely et al. 1966). The evaporation and rainfall data are annual averages, and the inflow is represented by three-year moving averages.

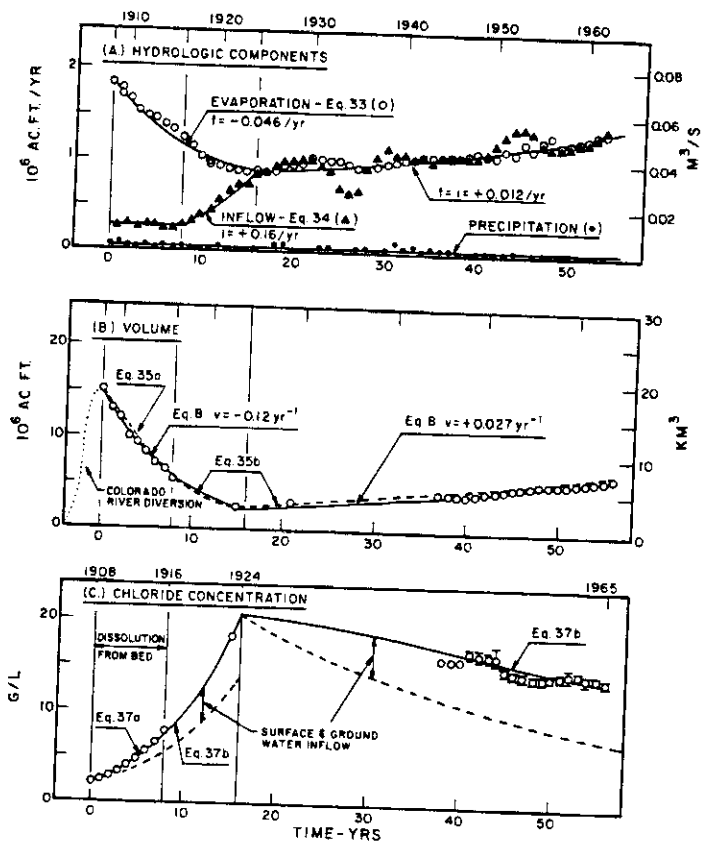


FIG. 3. Salton Sea: Hydrologic Components and TDS Concentration

The analysis is divided into three intervals: 1908–1916, 1916–1924, and 1924–1965. In the first and second intervals, the volume decreased primarily due to evaporation [Fig. 3(b)], with a commensurate increase in concentration of dissolved solids, expressed in terms of the chlorides [Fig. 3(c)]. In the third interval, the volume increased due to the precipitation and groundwater seepage, with the evaporation and inflow in balance, and the chloride concentration decreased.

The evaporation is approximated by an exponential function:

$$E = E_0 e^{ft} \dots \dots \dots (33)$$

The negative sign refers to the first two periods and the positive sign to the third period. Eq. 33 is shown in Fig. 3(a), with the inflow that is assigned as a constant in the first period and an exponential in the second and third periods:

$$I = I_0 e^{ut} \dots \dots \dots (34)$$

The outflow $Q = 0$. The rainfall and groundwater inflow are taken as constants for the entire period.

Eq. 33 is substituted in Eq. 1, the integration of which yields, for constant values of the remaining hydrologic components in the first interval

$$V = V_0 + (I + P + G)t - \frac{E_0}{f}(1 - e^{-fn}) \dots (35a)$$

For the second and third intervals, in which both Eqs. 33 and 34 apply

$$V = V_0 + (P + G)t + \frac{I_0}{i}(e^{in} - 1) \pm \frac{E_0}{f}(1 - e^{-fn}) \dots (35b)$$

In Eqs. 35a and b, 0 refers to the value at the beginning of each interval. The negative and positive signs of the evaporation term in Eq. 35b apply to the second and third periods, respectively. In the third period, the inflow equals the evaporation [Fig. 3(a)] and the increase in volume is due to the precipitation and groundwater [Fig. 3(b)].

The initial values of the volume V_0 and evaporation E_0 are 1.5×10^6 acre-ft (1.8 km^3) per year and 1.8×10^6 acre-ft (2.2 km^3) per year. The average precipitation P and groundwater inflow G are each 0.050×10^6 acre-ft (0.062 km^3) per year. The values of f and i are shown in Fig. 3. The volume at the end of the first interval provides the initial condition for the second, and similarly for the inflow and evaporation. The volume, calculated by Eqs. 35a and 35b, is presented as the dashed line in Fig. 3(b), a good approximation of which is provided by Eq. 8, as shown by the solid line. Thus, the basic differential equation for the mass balance is Eq. 12, with $\phi_1 = 0$ since the outflow is zero.

The analysis of the mass balance is divided into the same intervals as the hydrologic balance. The mass influx of dissolved solids is due to surface and groundwater inflow [Fig. 3(c)], with dissolution from the bed as an additional source during the first interval (Hely et al. 1966). Employing Eq. 13 for ϕ_2 , Eq. 12 becomes

$$\frac{dc}{dt} - vc = \frac{Ic_i + Gc_g}{V(t)} + D \dots (36)$$

in which c_i = surface-water concentration; c_g = groundwater concentration; and D = annual rate of chloride dissolution from the bed. Substituting Eq. 8 for $V(t)$ in Eq. 36 and simplifying, the solution for the first interval with constant inflow is

$$c = \left(\frac{c_i}{t_i} + \frac{c_g}{t_g} \right) t e^{iv} + \frac{D}{v}(e^{iv} - 1) + c_0 e^{iv} \dots (37a)$$

in which $t_i = V_0/I$; $t_g = V_0/G$, and c_0 = concentration at $t = 0$ (1908).

The solution for the second and third intervals, with exponentially increasing surface inflow and constant groundwater, is

$$c = \frac{c_i}{it_0} e^{iv}(e^{in} - 1) + \frac{c_g}{t_g} t e^{iv} + c_0 e^{iv} \dots (37b)$$

The negative and positive signs of v refer to the second and third intervals with decreasing and increasing volumes, respectively. The initial concentration c_0 is given by the solution of the preceding period. The hydrologic parameters v , i , t_0 , and t_g are established by the water balance, as previously defined in Eq. 37a.

The concentration in the surface inflow consists primarily of that in the

Alamo and New rivers, which contribute more than three-quarters of flow to Salton Sea during the period. A weighted average of the chloride concentration of this inflow was reported as 700 mg/L, rounded to the nearest hundred. The residual inflow consisted of drainage channels in the area, whose concentration c_i is taken as 800 mg/L, estimated from miscellaneous observations (Hely et al. 1966). The dissolution rate from the bed is approximated as follows: In 1907, the volume of the sea remained approximately constant with little inflow. The observed concentration increase of 220 mg/L in table 10 of Hely et al. (1966) is attributed to bed dissolution. Assuming this value as the initial rate and a linear decrease from 1908 to 1916, the average bed dissolution D over the period is 110 mg/L per year. The assignment of the groundwater concentration, for which no information was available, is a degree of freedom. Assuming this concentration is in the order of thousands, a value of c_g of 3,000 mg/L, representative of similar regions ("Mono Basin Geology" 1987), closes the mass balance.

Given these values, the concentration, calculated in accordance with Eqs. 37a and 37b, is presented in Fig. 3(c). The most significant factor is the concentrating and diluting effect of the initial concentration by the change in volume, as shown by the dashed lines. The rise in concentration in 1924 is due primarily to the evaporation and the subsequent decrease due to the rainfall and groundwater, with the inflow and evaporation in balance. The alternate approach of calculating the mass and dividing by the volume yields the concentration within 1–2% of that calculated previously. This procedure is used in the following examples.

The preceding analysis adequately reproduces the long-term trend in concentration. A more refined model, however, is required for the variation of the trend from 1945 to 1965 [Fig. 3(c)]. This period is divided into four intervals [Fig. 4(a)], in which the inflow is more realistically defined as shown by the solid line. In the first and third intervals, hydrologic equilibrium prevails and the volume is constant, identified by V_1 and V_3 in Fig. 4(b). In the second and fourth intervals, rainfall, groundwater, and inflow in excess of the evaporation, the latter depicted by the dash-dot line in Fig. 4(a), are included in the hydrologic balance. The inflow excess, I_a , is expressed by a sine function, and Eq. 1 becomes

$$\frac{dV}{dt} = P + G + I_a \sin \omega t \dots\dots\dots (38)$$

integration of which yields for the second and fourth intervals

$$V = V_0 + (P + G)t + \frac{I_a}{\omega} (1 - \cos \omega t) \dots\dots\dots (39)$$

in which V_0 = volume at $t = 0$ for each interval; I_a = flow amplitude; ω = frequency = $2\pi/p$; and p = period. $V_{01} = 4.0 \times 10^6$ acre-ft (4.9 km^3), and P and G have the same values as in the long-term analysis. The inflow amplitudes are 0.28 and 0.10×10^6 acre-ft (0.34 and 0.12 km^3) per year respectively, each with a period of 10 years. The volume, calculated in accordance with these conditions, is shown by the solid line in Fig. 4(b) and compared with that of the preceding analysis, shown by the dashed line.

The basic differential equation for the mass balance is Eq. 11, with $\phi_1 = 0$ since the outflow is zero. The mass inputs include the contributions of the surface inflow and groundwater, both of which are constant in the first and third intervals. Integration of Eq. 11 for these conditions yields

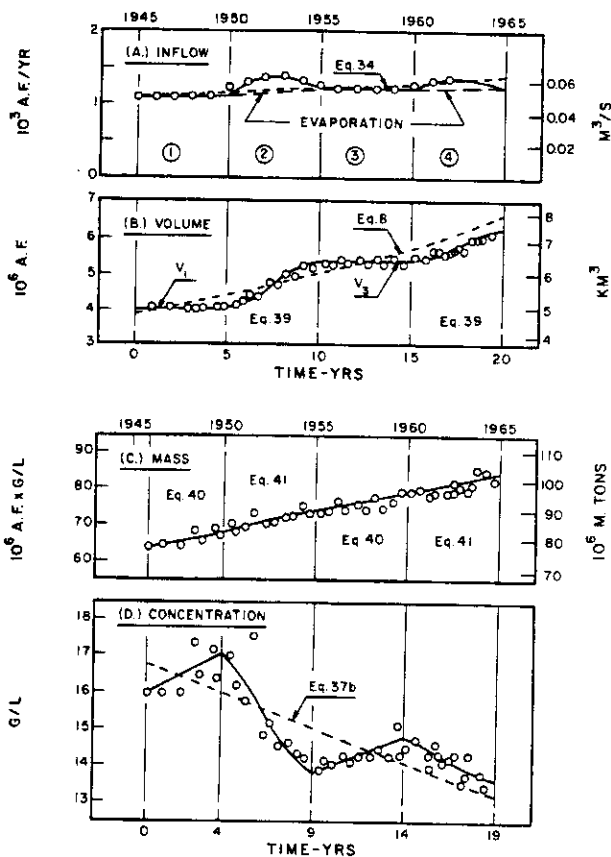


FIG. 4. Salton Sea: TDS Concentration

$$M = M_0 + I_0 t + G_c t \dots \dots \dots (40)$$

Surveys were reinitiated in 1945, and the first measurement of concentration (16,000 mg/L) was taken in the latter part of December. Therefore the mass calculations are initiated on January 1, 1946, with M_0 at $64,000 \times 10^6$ acre-ft-mg/L (7.9×10^6 T).

For the second and fourth intervals, with the excess surface inflow and groundwater, as defined by the hydrologic balance (Eq. 39) and the surface inflow, equal to the evaporation [Fig. 4(a)], the solution is

$$M = M_0 + \frac{I_0 c_i}{i} (e^{it} - 1) + \frac{I_0 c_i}{\omega} (1 - \cos \omega t) + G_c t \dots \dots \dots (41)$$

The initial conditions for the subsequent intervals are established from the hydrologic and mass balances of the preceding intervals. The mass calculated by Eqs. 40 and 41 is presented in Fig. 4(c) with the observed values. The concentration follows from the mass and volume, as shown in Fig. 4(d). Also shown for comparison is the concentration from the previous analysis (Eq. 37b).

It is interesting to note that in the period following 1965, subsurface agricultural return from Imperial Valley resulted in serious flooding of residential and recreational areas of Salton Sea. In addition, the heavy salt content of the runoff caused an increase in the salinity of Salton Sea which conservationists feared would endanger its abundant supply of fish (Holles 1977).

Cheney Reservoir

Cheney Reservoir is located on the north branch of the Ninnescan River, a tributary of the Arkansas River in Kansas. The construction of the reservoir was completed in 1964, and water was first withdrawn in May 1965 to supply the city of Wichita, Kansas. A survey was conducted by Ward and Karak (1969) from mid-1965–September 1967 in order to determine the effect of impoundment on water quality. The hydrologic data on river inflow I , outflow Q , rainfall P , and evaporation E are presented as histograms of average monthly values and three-month running averages in Fig. 5.

The components of the water balance in the 1965–66 period are formulated as follows:

$$I = I_m + I_a \sin \omega_1 t \dots \dots \dots (42)$$

$$Q = Q_m + Q_a \sin \omega_2 t \dots \dots \dots (43)$$

$$P = P_0 e^{-\tau} - P_a \sin \omega_3 t \dots \dots \dots (44)$$

$$E = E_m + E_a \cos \omega_4 t \dots \dots \dots (45)$$

where m and a = the mean and amplitude values, respectively, of the periodic functions of frequency ω ; and 0 = the initial value of exponential function. In 1967, average values are assigned to represent the various components, as shown in Fig. 5.

Substituting Eqs. 42, 43, 44, and 45 in Eq. 1 and integrating yields for the period from mid-1965 through 1966

$$V_1 = V_{01} + I_m t + \frac{I_a}{\omega_1} (1 - \cos \omega_1 t) - Q_m t - \frac{Q_a}{\omega_2} (1 - \cos \omega_2 t) \frac{P_0}{r} (1 - e^{-\tau}) + \frac{P_a}{\omega_3} (\cos \omega_3 t - 1) - E_m t - \frac{E_a}{\omega_4} (\sin \omega_4 t) \dots \dots \dots (46)$$

in which V_{01} = 80,200 acre-ft (0.099 km³) at $t = 0$ (July 1, 1965).

The calculated volume at the end of the first interval provides the initial condition V_{02} for the second interval (January–August 1967), which, for constant inflows and outflows, is

$$V_2 = V_{02} + [(I + R) - (Q + E)]t \dots \dots \dots (47)$$

The volume, calculated in accordance with Eqs. 46 and 47, is shown in Fig. 6(a) and its components in Fig. 6(b). Note the magnitude and variability of the outflow, both of which are small relative to those of the other components. The assumption of a constant outflow, as indicated by the dashed line, yields essentially the same calculated volume (<1% difference). By contrast, a similar approximation of the inflow, which is the most significant component of the hydrologic balance, would result in a much greater difference between the calculated and observed values of the volume. These observations are also relevant to the mass balance. Since the chloride concentration of the inflow and outflow are of the same order (-200 mg/L), it follows from Eq. 10 or Eq. 11 that the mass rate of change associated with the

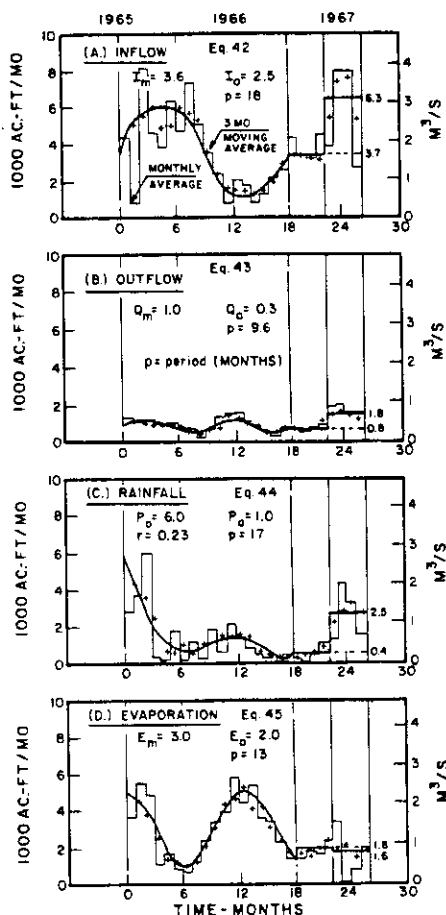


FIG. 5. Cheney Reservoir: Hydrologic Components

inflow, is more significant than that of the outflow, as indicated by the cumulative values in Fig. 6.

The average monthly values of the volume are presented in Fig. 7(a). Fig. 7(b) is an enlarged scale of the cumulative outflow, the slope of which, shown by the solid lines, defines the average outflow for each of the four periods. The residence times, based on the volumes and average outflows, are presented in Fig. 7(c). The exponential function (Eq. 17) defines the residence, as indicated by the solid lines, with the associated coefficients. The dashed line is the average for the entire period.

For the periodic inflow function, representative of 1965 and 1966 (Eq. 42), the mass equation, in accordance with the exponential residence (Eq. 19), is

$$\frac{dM}{d\tau} \pm \frac{M}{at_{\infty}} = \frac{\left[I_m + A_o \sin \left(\frac{\omega_1}{a} \ln \tau \right) \right]}{a\tau} c_l \dots \dots \dots (48)$$

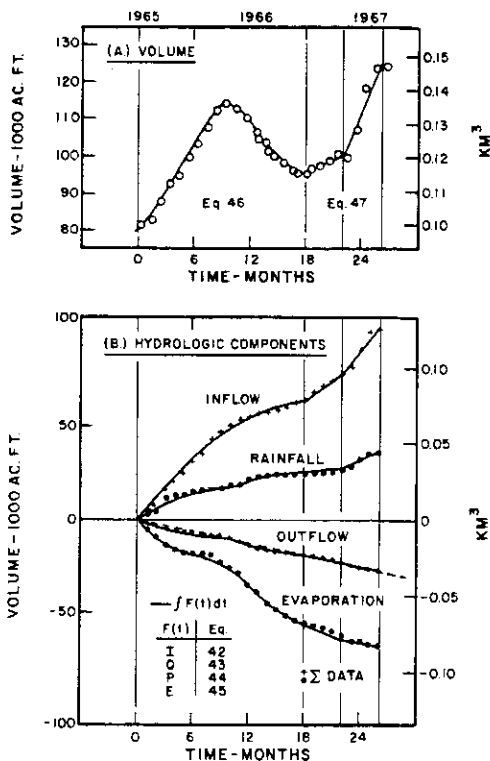


FIG. 6. Cheney Reservoir: Total Volume and Component

The solution, based on the approximation leading to Eq. 24, is, for $\omega_1 t_d \gg 1$:

$$M^{+\phi} = M_0 + I_m c_i \bar{t}_d (e^{I_d} - 1) + I_a c_i \bar{t}_d \left(\frac{1 - e^{I_d} \cos \omega_1 t_d}{\omega_1 \bar{t}_d} \right) \dots (49)$$

in which M_0 = mass at $t = 0$; c_i = chloride concentration of the inflow; and ϕ is given by Eqs. 25a and 26a, and t_d by Eqs. 25b and 26b. The first nine months of the 1965-66 period is characterized by an increase in residence (Eqs. 26a and b) and the second nine months by a decreasing residence (Eqs. 25a and b). In 1967, each of the intervals is characterized by constant inflow and exponential residence. Eqs. 48 and 49 apply with $I_a = 0$.

The initial mass ($c_0 = 130$ mg/L) is 10.4×10^6 acre-ft-mg/L (12.8×10^6 T). The inflow parameters, I_m , I_a , and ω , are given in Fig. 5 and the exponent a in Fig. 7. The mass influx of solids to Cheney Reservoir is due to the Ninnescan River. Measurements of electrical conductivity were made at an upstream river location, and total dissolved solids and chlorides were made at a number of stations in the reservoir. Based on a correlation between the chlorides and electrical conductivity, the chloride concentration of the river inflow is 210 mg/L for the first two intervals, 220 mg/L for the third, and 180 mg/L for the fourth, in accordance with the inverse relations between river flow and dissolved solids (O'Connor 1976). Fig. 8(a) presents

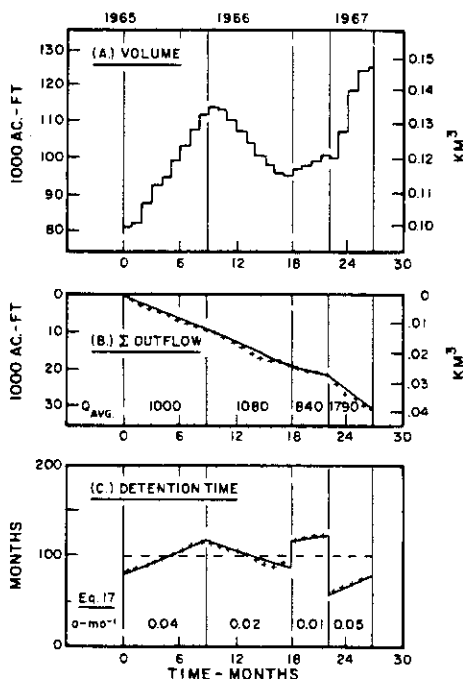


FIG. 7. Cheney Reservoir: Residence Time

the mass (Eq. 49) and its components. Dividing by the volume (Eqs. 46 and 47) yields the concentration, as shown in Fig. 8(b), with the average monthly values of the chloride and TDS data. The chloride concentrations were considered the most accurate of the various constituents measured (Ward and Karak 1969). Typical ranges of the observed concentration are indicated, reflecting the spatial variation within the reservoir and the temporal variation within the month. The minimum range occurs during the winter and early spring and the maximum during stratification in late summer.

The dashed line in Fig. 7(c) represents the average residence time for the entire period ($t_d = 100$ months), for which the basic mass-balance equation with the periodic inflow is

$$\frac{dM}{dt} + \frac{M}{t_d} = (I_m + I_a \sin \omega_1 t)c_i \dots \dots \dots (50)$$

The solution is for $\omega_1 t \gg 1$:

$$M = I_m c_i \bar{t}_d (1 - e^{-t/\bar{t}_d}) + I_a c_i \bar{t}_d \left(\frac{e^{-t/\bar{t}_d} - \cos \omega_1 t}{\omega_1 \bar{t}_d} \right) + M_0 e^{-t/\bar{t}_d} \dots \dots \dots (51)$$

For the third and fourth intervals, Eq. 51 applies with $I_a = 0$. The concentration, based on Eq. 51, is also shown by the dashed line in Fig. 8(b). Due to the large detention time, by contrast to the time scale of the analysis, the difference between the two calculated profiles is less than 5%, well within the range of the data. This point is also borne out by the mass outflow, as

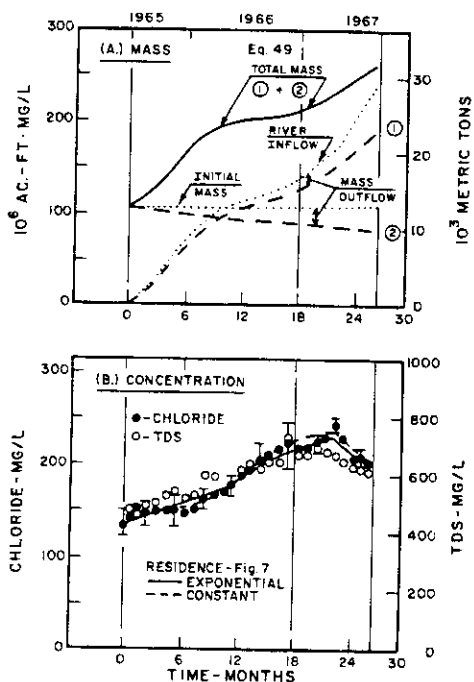


FIG. 8. Cheney Reservoir: Mass Concentration of Chlorides

shown in Fig. 8(a), with which the residence time is associated in the mass balance (Eqs. 48 and 50).

Mono Lake

Mono Lake, in eastern California, is a closed lake in the Great Basin. Water for municipal use and irrigation has been diverted from the basin since 1941, causing a decrease in volume and increase in salinity ("Report" 1979; "Mono Basin Ecosystem" 1987; "Mono Basin Geology" 1987), as shown in Fig. 9. High runoff in the late 1960s compensated for the diversion and evaporation, resulting in an approximate equilibrium, as indicated, and higher runoff in the early 1980s, increased the volume with a commensurate decrease in salinity.

The periods of hydrologic equilibrium provide a basis for estimating the net loss of water from the lake (evaporation minus precipitation) by the steady-state solution of Eq. 1, yielding an average value of 3.5 ft (1.1 m) per year. The gauged runoff comprises about 80% of the total ("Mono Basin Ecosystem" 1987), from which is subtracted the diversion ("Report" 1979), to obtain the net inflow to the lake, indicated in Fig. 9(a). The depth-volume relation is approximated by Eq. 2 with $m = 2$, for the interval from 1947 through 1985 [$V_0 = 4.25 \times 10^6$ acre-ft (5.23 km³), $A_0 = 56 \times 10^3$ acres (227 km²), $H_0 = 76$ ft (23 m)]. The water-level elevation, surface area, and volume, calculated by Eqs. 5a and 5b, are shown in Figs. 9(a) and 9(b).

If the mass in the lake is constant, as assumed in "Mono Basin Ecosystem" (1987), the concentration varies inversely with the volume, as shown

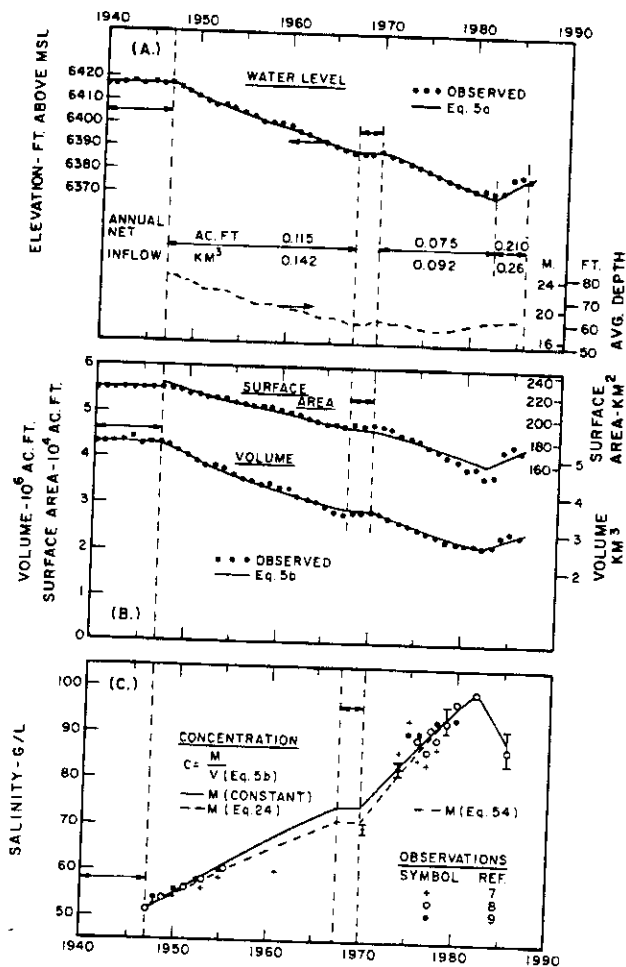


FIG. 9. Mono Lake: Elevation, Volume, and Salinity

by the solid line in Fig. 9(c). The calculated concentrations are in general agreement with the observations from 1976 to 1985, but greater than the observed concentrations in the earlier years, particularly in 1961 and 1971 ("Report" 1979). The deviation may be reconciled to some degree by assuming a diffusional flux of salt to the bed or flow to groundwater storage from the lake in the first period and from the groundwater or seepage area to the lake in the second and third periods (Fig. 5) ("Mono Basin Ecosystem" 1987). The latter further requires a smaller inflow [0.14×10^6 acre-ft (0.17 km^3) per year], which is consistent with the extreme drought recorded in 1975-77. The mass balance in the first period, with flow from the lake to the groundwater reservoir, is

$$\frac{dM}{dt} = I_c - G_c \dots \dots \dots (52)$$

Gc may also represent the salt flux to the bed. Introducing the volume, approximated by the exponential function from Eq. 8, in the groundwater component, Eq. 52 becomes

$$\frac{dM}{dt} + \frac{e^{-\lambda t}}{v t_{d0}} M = Ic, \dots \dots \dots (53)$$

which is identical to Eq. 18. The solution is therefore Eq. 24 with $W = Ic$; $a = v$; and $t_{d0} = V_0/G$. The exponential approximation of the volume yields $v = 0.019 \text{ yr}^{-1}$ and $t_{d0}^{-1} = 0.0024 \text{ yr}^{-1}$. The surface inflow is also included for the sake of generality, although its contribution ($c_i = 0.1 \text{ gr/l}$) is insignificant (<1%).

For the latter periods (1970–85), the mass balance with groundwater inflow to the lake is simply

$$M = M_0 + Ic_t + Gc_g t \dots \dots \dots (54)$$

The groundwater concentration c_g is taken as 25 g/L, the average of the values in the seepage zone of the western region of the lake. The mass (Eqs. 24 and 54) divided by the volume (Eq. 5a and b) yields the concentration, shown by the dashed line in Fig. 9. An additional source of salt may be due to wind shear or stream erosion of the exposed land surfaces during this period ("Mono Basin Ecosystem" 1987).

Some degree of confirmation of the values of the evaporation and, to a lesser extent, of the precipitation and inflow are provided by radionuclides, which have been utilized as tracers for hydrologic research (Stout 1967). The observed concentrations of an isotope (tritium) are compared to those calculated in accordance with the hydrologic balance developed.

The atmospheric concentration of tritium at Portland, Oregon (Herczeg and Imboden 1988), is presented in Fig. 10(a), which is defined by an exponential function, similar to Eq. 28, with the coefficients indicated for each period. This distribution is identical to that employed in the analysis of plutonium in Lake Michigan (O'Connor 1988). These data, which are assumed to be representative of the Mono Lake region, permit the determination of the mass influx due to precipitation and surface inflow. The groundwater is a source or sink of mass, consistent with the salinity analysis.

In the interval from 1962 to 1969, when a loss of mass to the groundwater is assumed as in the chloride analysis, the mass balance is

$$\frac{dM}{dt} = (I + P)c_{d0}e^{-\lambda t} - Gc - \Psi \epsilon A c - K_d V c \dots \dots \dots (55)$$

in which c_{d0} = initial concentration; Ψ = isotropic fractionation; ϵ = unit evaporation; and K_d = decay coefficient. Introducing the volume in the groundwater and evaporation terms, Eq. 55 becomes

$$\frac{dM}{dt} + K_T M = W_0 e^{-\lambda t} \dots \dots \dots (56)$$

in which $K_T = K_d + \Psi \epsilon / H + 1/t_d$; $H = A/V$; $t_d = V/G$; and $W_0 = (I + P)c_{d0}$.

Eq. 56 is of the general form of Eq. 18, due to the decreasing volume and depth. The depth changes from 18 m to 17 m over the interval and the order of t_d^{-1} is 0.002 yr^{-1} , compared with $K_T = 0.13 \text{ yr}^{-1}$ [$K_d = 0.05 \text{ yr}^{-1}$, $\epsilon = 4 \text{ ft (1.2 m) per year}$, $\Psi = 0.9$]. Therefore, the assumption of average values introduces no significant error (~1%), and the solution of Eq. 56 is

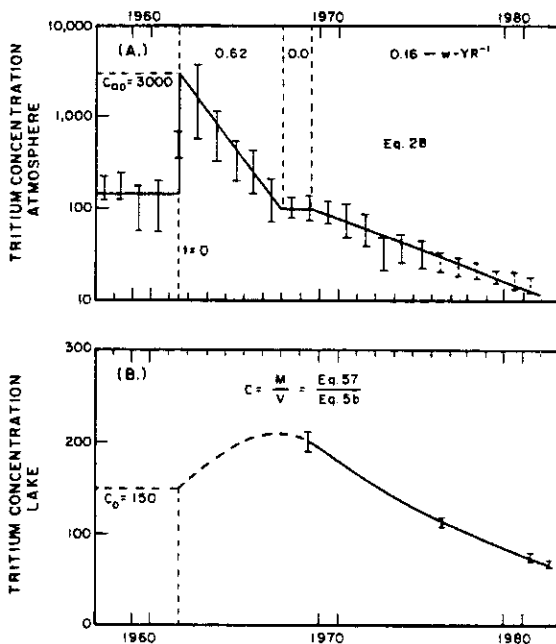


FIG. 10. Mono Lake: Tritium Concentration

$$M = \frac{W_0}{K_T - w} (e^{-wt} - e^{-K_T t}) + M_0 e^{-K_T t} \quad (57)$$

Since the groundwater effect is insignificant ($<0.1\%$), it is not included in the analysis of the subsequent years, for which Eq. 57 holds with $t_a^{-1} = 0$. The analysis is initiated in mid-1962 with $C_{a0} = 150$, the residue of bomb testing in the previous decade. The assumption that this concentration is also representative of the inflow may compensate for the enrichment factor (~ 1.5) of the atmospheric loading, recently reported in a comparable analysis (Herzegg and Imboden 1988). The inflows and precipitation are those computed by the hydrologic balance, already described in the salinity analysis. The mass, Eq. 57, divided by the volume, Eq. 5b [Fig. 9(b)], yields the concentration, as shown in Fig. 10(b).

SUMMARY AND CONCLUSIONS

Analytical solutions of the hydrologic- and mass-balance equations have been developed to define the long-term change in water volume and dissolved solids. The calculated trends and values are in reasonable agreement with the historical observations of water levels, volumes, and concentrations in lakes and reservoirs. The equations may therefore provide an additional tool in the analysis of water-quality problems, similar to those reported in this paper. The groundwater flow and concentration, for which data are usually most limited, are the most uncertain parameters in the analysis. The error introduced in the mass outflow, by neglecting the effect of stratification, is minimal in the examples presented but may be more significant in

other applications, particularly for reservoirs.

Further development of the equations is planned to incorporate the effect of stratification with decreasing freshwater volumes, anticipated with the long-term trend of increasing atmospheric temperatures. This phenomenon will have a dual effect on water quality. On the average annual basis, the decrease in volume will generally result in an increase in concentration of contaminants and, potentially more important on the seasonal time scale, will intensify water-quality problems associated with benthic fluxes. Given further substantiation, the equations and the general approach may be used to project future conditions and to evaluate management alternatives in lakes and reservoirs.

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